

Chapter 14: Probability

NCERT · Class 11 · Maths

Exercise 14.1

Question 1

A die is rolled. Let E be the event “die shows 4” and F be the event “die shows even number”. Are E and F mutually exclusive?

Solution

Ans: 1,2,3,4,5 and 6 are the possible outcomes when the die is thrown.

Therefore, $S = \{1,2,3,4,5,6\}$

According to the question,

E is event die shows 4

$$E = (4)$$

F is die shows even number

$$F = (2, 4, 6)$$

$$E \cap F = (4) \cap (2, 4, 6)$$

$$E \cap F = 4$$

Therefore, we can conclude that E and F are not mutually exclusive event.

Question 2

A die is thrown. Describe the following events:

(i) A: a number less than 7.

(ii) B: a number greater than 7.

(iii) C: a multiple of 3

(iv) D: a number less than 4.

(v) E: an even number greater than 4.

(vi) F: a number not less than 3.

Solution

Ans:

$$1, 2, 3, 4, 5$$

and 6 are the possible outcomes when the die is thrown.

Therefore,

$$S = (1, 2, 3, 4, 5, 6)$$

According to the question,

A: a number less than 7

All the numbers in the die are less than 7,

$$A = (1, 2, 3, 4, 5, 6)$$

Ans:

$$1, 2, 3, 4, 5$$

and 6 are the possible outcomes when the die is thrown.

Therefore,

$$S = (1, 2, 3, 4, 5, 6)$$

According to the question,

B: a number greater than 7

There is no number on dice greater than 7.

$$B = \phi$$

Ans:

$$1, 2, 3, 4, 5$$

and 6 are the possible outcomes when the die is thrown.

Therefore,

$$S = (1, 2, 3, 4, 5, 6)$$

According to the question,

C : a number multiple of 3

Multiple of 3 between 1 to 6 is 3 and 6

Therefore,

$$C = (3, 6)$$

Ans: $\{1, 2, 3, 4, 5\}$ and 6 are the possible outcomes when the die is thrown.

Therefore,

$$S = (1, 2, 3, 4, 5, 6)$$

According to the question,

D: a number less than 4

$$D = (1, 2, 3)$$

Ans:

$$1, 2, 3, 4, 5$$

and 6 are the possible outcomes when the die is thrown.

Therefore,

$$S = (1, 2, 3, 4, 5, 6)$$

According to the question,

E: an even number greater than 4

There are only one number which is even greater than 4.

Therefore,

$$E = (6)$$

Ans: Let us 1,2,3,4,5 and 6 are the possible outcomes when the die is thrown.

Therefore,

$$S = (1, 2, 3, 4, 5, 6)$$

According to the question,

F: a number not less than 3

$$F = (3, 4, 5, 6)$$

Also find

$$A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, D - -E, A - -C, E \cap F', F'$$

Ans: As we have to find,

$$A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, D - -E, A - -C, E \cap F', F'$$

Therefore,

$$A \cap B = (1, 2, 3, 4, 5, 6) \cap (\phi)$$

$$A \cap B = (\phi)$$

$$B \cup C = (\phi) \cup (3, 6)$$

$$B \cup C = (3, 6)$$

$$E \cap F = (6) \cap (3, 4, 5, 6)$$

$$E \cap F = (6)$$

$$D \cap E = (1, 2, 3) \cap (6)$$

$$D \cap E = (\phi)$$

$$D - -E = (1, 2, 3) - - (6)$$

$$D - -E = (1, 2, 3)$$

$$A - -C = (1, 2, 3, 4, 5, 6) - - (3, 6)$$

$$A - -C = (1, 2, 4, 5)$$

$$F' = S - -F$$

$$F' = (1, 2, 3, 4, 5, 6) - - (3, 4, 5, 6)$$

$$F' = (1, 2)$$

$$E \cap F' = (6) \cap (1, 2)$$

$$E \cap F' = (\phi)$$

Question 3

An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8,

B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Solution

Ans: Let

$$1, 2, 3, 4, 5$$

and 6 are the possible outcomes when the die is thrown.

As in this question pair of die is thrown, so sample space will be,

$$S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$$

A: The sum is greater than 8

$$A = (3, 6), (4, 5), (4, 6), (5, 4), (5, 5)$$

$$(5, 6), (6, 3), (6, 4), (6, 5), (6, 6)$$

B: 2 occur on the die

$$B = (2, 1), (2, 2), (2, 3), (2, 4), (2, 5)$$

$$(1, 2), (3, 2), (4, 2), (5, 2), (6, 2), (2, 6)$$

C: the sum is at least 7 and multiple of 3

$$C = \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$$

Now, we can find pairs of the events which are mutually exclusive or not.

(i)

$$A \cap B = \phi$$

As, there is no common element in A and B

Therefore, A and B are mutually exclusive

(ii)

$$B \cap C = \phi$$

Since there is no common element between

Therefore, B and C are mutually exclusive.

(iii)

$$A \cap C = \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$$

$$A \cap C \neq \phi$$

As, A and C have common elements.

Therefore A and C are not mutually exclusive.

Question 4

Three coins are tossed once. Let A denote the event 'three heads show', B denote the event "two heads and one tail show", C denote the event" three tails show" and D denote the event"a head shows on the first coin". Which events are

(i) Mutually exclusive? (ii) Simple? (iii) Compound?

Solution

Ans: As three coins are tossed once so the possible sample space will be,

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTH\}$$

According to question,

A: 'three heads'

$$A = (HHH)$$

B: two heads and one tail

$$B = (HHT, THH, HTH)$$

C :three tails

$$C = (TTT)$$

D: a head shows on the first coin

$$D = (HHH, HHT, HTH, HTT)$$

(i) Mutually Exclusive

$$A \cap B = (HHH) \cap (HHT, THH, HTH)$$

$$A \cap B = \phi$$

Therefore, A and B are mutually exclusive.

$$A \cap C = (HHH) \cap (TTT)$$

$$A \cap C = \phi$$

Therefore, A and C are mutually exclusive.

$$A \cap D = (HHH) \cap (HHH, HHT, HTH, HTT)$$

$$A \cap D = (HHH)$$

$$A \cap D \neq \phi$$

Therefore, A and D are not mutually exclusive

$$B \cap C = (HHT, HTH, THH) \cap (TTT)$$

$$B \cap C = \phi$$

Therefore, B and C are mutually exclusive.

$$B \cap D = (HHT, THH, HTH) \cap (HHH, HHT, HTH, HTT)$$

$$B \cap D = (HHT, HTH)$$

$$B \cap D \neq \phi$$

Therefore, B and D are not mutually exclusive.

$$C \cap D = (TTT) \cap (HHH, HHT, HTH, HTT)$$

$$C \cap D = \phi$$

Therefore, C and D are mutually exclusive.

(ii) Simple Event

If an event has only one outcome then it is called a simple event.

$$A = (HHH)$$

$$C = (TTT)$$

Both A and C have only one element,

Therefore, they are simple events.

(iii) Compound Events

If an event has more than one outcome then it is called a Compound event

$$B = (HHT, HTH, THH)$$

$$D = (HHH, HHT, HTH, HTT)$$

Both B and D have more than one element,

Therefore, they are compound events.

Question 5

Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

As three coins are tossed once so the possible sample space contains,

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Two events which are mutually exclusive.

Let A be the event of getting only head

$$A = \{HHH\}$$

Let B be the event of getting only tail

$$B = \{TTT\}$$

Therefor,

$$A \cap B = \phi$$

So, A and B are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

As three coins are tossed once so the possible sample space contains,

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Three events which are mutually exclusive and exhaustive

Now,

Let P be the event of getting exactly two tails

$$P = \{HTT, TTH, THT\}$$

Let Q be the event of getting at least two heads

$$Q = \{HHT, HTH, THH, HHH\}$$

Let R be the event of getting three tail

$$C = \{TTT\}$$

$$P \cap Q \cap R = \{HTT, TTH, THT\} \cap \{HHT, HTH, THH, HHH\} \cap \{TTT\}$$

$$P \cap Q \cap R = \phi$$

Therefore, P, Q and R are mutually exclusive.

And also,

$$P \cup Q \cup R = \{HTT, TTH, THT\} \cup \{HHT, HTH, THH, HHH\} \cup \{TTT\}$$

$$P \cup Q \cup R = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$P \cup Q \cup R = S$$

Hence P, Q and R are exhaustive events.

(iii) Two events, which are not mutually exclusive.

As three coins are tossed once so the possible sample space contains,

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Two events, which are not mutually exclusive

Let A be the event of getting at least two heads

$$A = \{HHH, HHT, THH, HTH\}$$

Let B be the event of getting only head

$$B = \{HHH\}$$

$$A \cap B = \{HHH, HHT, THH, HTH\} \cap \{HHH\}$$

$$A \cap B = \{HHH\}$$

$$A \cap B \neq \phi$$

Therefore, A and B are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

As three coins are tossed once so the possible sample space contains,

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Two events which are mutually exclusive but not exhaustive

Let P be the event of getting only Head

$$P = \{HHH\}$$

Let Q be the event of getting only tail

$$Q = \{TTT\}$$

$$P \cap Q = \{HHH\} \cap \{TTT\}$$

$$P \cap Q = \phi$$

Therefore, P and Q are mutually exclusive events.

But,

$$P \cup Q = \{HHH\} \cup \{TTT\}$$

$$P \cup Q = \{HHH, TTT\}$$

$$P \cup Q \neq S$$

Therefore, P and Q are not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive.

As three coins are tossed once so the possible sample space contains,

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Three events which are mutually exclusive but not exhaustive

Let X be the event of getting only head

$$X = \{HHH\}$$

Let Y be the event of getting only tail

$$Y = \{TTT\}$$

Let Z be the event of getting exactly two heads

$$Z = \{HHT, THH, HTH\}$$

Now,

$$X \cap Y \cap Z = \{HHH\} \cap \{TTT\} \cap \{HHT, THH, HTH\}$$

$$X \cap Y \cap Z = \phi$$

Therefore, they are mutually exclusive

Also

$$X \cup Y \cup Z = \{HHH\} \cup \{TTT\} \cup \{HHT, THH, HTH\}$$

$$X \cup Y \cup Z = \{HHH, TTT, HHT, THH, HTH\}$$

$$X \cup Y \cup Z \neq S$$

So, X, Y and Z are not exhaustive.

Therefore, X, Y and X are mutually exclusive but not exhaustive.

Solution

Solution coming soon.

Question 6

Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5 .

Describe the events:

(i)

$$A'$$

(ii) not B

(iii) A or B

(iv) A and B

(v) A but not C

(vi) B or C

(vii) B and C

(viii)

$$A \cap B' \cap C'$$

Solution

Ans: Let 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question is given that pair of die is thrown, so sample space will be,

$$S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$$

A: Getting an even number on first die

$$A = (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

B: Getting odd number on first die

$B = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$

C: Getting the sum of the number on dice

≤ 5

$C = (1, 1), (1, 2), (1, 3), (1, 4), (2, 1)$

$(2, 2), (2, 3), (3, 1), (3, 2), (4, 1)$

Ans:

$A = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$

Ans:

$B = (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$

$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

Ans:

$A \text{ or } B (A \cup B)$

=
 $(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$

$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$

$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

Ans:

$A \text{ and } B (A \cap B) = \phi$

Ans:

$A \text{ but not } C (A - C)$

=
 $(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 2), (4, 5)$

$(4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

Ans:

$B \text{ or } C (B \cup C)$

=
 $(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$

$(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)$

$(3, 4), (3, 5), (3, 6), (4, 1), (5, 1), (5, 2)$

$(5, 3), (5, 4), (5, 5), (5, 6)$

Ans:

$$B \text{ and } C \quad (B \cap C) = \{(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)\}$$

Ans:

$$A \cap B' \cap C' = A \cap A \cap C'$$

$$A \cap A \cap C' = A \cap C'$$

$$A \cap C' = \{(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

Question 7

Refer to question 6 above, state true or false: (give reason for your answer)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii)

$$A = B'$$

(iv) A and C are mutually exclusive

(v) A and

$$B'$$

are mutually exclusive.

(vi)

$$A', B', C$$

are mutually exclusive and exhaustive.

Solution

Ans: Let 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question is given that pair of die is thrown, so sample space will be,

$$S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

A: Getting an even number on first die

$A = (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$

$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

B: Getting odd number on first die

$B = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$

C: Getting the sum of the number on dice

$$\leq 5$$

$C = (1, 1), (1, 2), (1, 3), (1, 4), (2, 1)$

$(2, 2), (2, 3), (3, 1), (3, 2), (4, 1)$

Ans:

$$(A \cap B) = \phi$$

Therefore, A & B are mutually exclusive.

Ans:

$$(A \cap B) = \phi$$

Therefore, A & B are mutually exclusive.

$$A \cup B$$

$$= (1, 1)(1, 2)(1, 3), (1, 4), (1, 5), (1, 6)$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$$

$$A \cup B = S$$

Therefore, A and B are exhaustive also.

Ans:

$$A = (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$$

$$B' = (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$$

Therefore,

$$A = B'$$

Ans: As we have,

$$A \cap C = \{(2, 1), (2, 2), (2, 3), (4, 1)\}$$

$$A \cap C \neq \phi$$

Therefore, A and C are not mutually exclusive

Ans: As we have,

$$A \cap B' = A \cap A = A$$

$$\therefore A \cap B' \neq \phi$$

Therefore, A and

$$B'$$

are not mutually exclusive.

Ans:

$$A' = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$$

$$B' = (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$$

$$C = (1, 1), (1, 2), (1, 3), (1, 4), (2, 1)$$

$$(2, 2), (2, 3), (3, 1), (3, 2), (4, 1)$$

$$A' \cap B' = \phi$$

But,

$$B' \cap C = \{(2, 1), (2, 2), (2, 3)\}$$

$$B' \cap C \neq \phi$$

$$A' \text{ and } B'$$

are mutually exclusive.

B' and C

are not mutually exclusive.

Therefore,

A', B', C

are not mutually exclusive.

$A' \cup B' \cup C$

- =
- (1, 1)(1, 2)(1, 3), (1, 4), (1, 5), (1, 6)
- (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
- (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
- (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
- (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
- (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$A' \cup B' \cup C = S$

Therefore,

A', B', C

are exhaustive.

Exercise 14.2

Question 1

Which of the following cannot be valid assignment of probabilities for outcomes of sample space

$S = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7)$

?

Assignment

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
------------	------------	------------	------------	------------	------------	------------	------------

(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	1/7	1/7	1/7	1/7	1/7	1/7	1/7
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	1/14	2/14	3/14	4/14	5/14	6/14	15/14

Solution

Ans:

(a) Probability should be less than 1 and positive.

$$P(E) = 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6$$

$$P(E) = 1$$

Therefore, the given assignment is valid.

b) Probability should be less than 1 and positive.

$$P(E) = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$$

$$P(E) = 1$$

Therefore, the given assignment is valid.

c) Probability should be less than 1 and positive.

$$P(E) = 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$$

$$P(E) = 2.8 > 1$$

Here probability is greater than 1.

Therefore, the given assignment is not valid.

d) Probability can never be negative.

Therefore, the assignment is not valid.

e) Probability should be less than 1 and positive.

$$P(E) = \frac{1}{14} + \frac{2}{14} + \frac{3}{14} + \frac{4}{14} + \frac{5}{14} + \frac{6}{14} + \frac{15}{14}$$

$$P(E) = \frac{28}{14} > 1$$

Therefore, the assignment is not valid.

Question 2

A coin is tossed twice, what is the probability that at least one tail occurs?

Solution

Ans: Here coin is tossed twice, then sample space is

$$S = (TT, HH, TH, HT)$$

Number of possible outcomes

$$n(S) = 4$$

Let A be the event of getting at least one tail

$$\therefore n(A) = 3$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{4}$$

Question 3

A die is thrown, find the probability of following events:

- (i) A prime number will appear,
- (ii) A number greater than or equal to 3 will appear,
- (iii) A number less than or equal to one will appear,
- (iv) A number more than 6 will appear,
- (v) A number less than 6 will appear.

Solution

Ans: Here die is thrown, therefore the sample space of outcomes will be

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let A be the event of getting a prime number,

$$A = \{2, 3, 5\}$$

Then,

$$n(A) = 3$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{6}$$

$$P(A) = \frac{1}{2}$$

Ans: Let B be the event of getting a number greater than or equal to 3,

$$B = \{3, 4, 5, 6\}$$

Then,

$$n(B) = 4$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(B) = \frac{4}{6}$$

$$P(B) = \frac{2}{3}$$

Ans: Let A be the event of getting a number less than or equal to 1,

$$A = \{1\}$$

Then,

$$n(A) = 1$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{6}$$

Ans: Let A be the event of getting a number more than 6, then

$$A = \{0\}$$

Then,

$$n(A) = 0$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{0}{6}$$

$$P(A) = 0$$

Ans: Let E be the event of getting a number less than 6, then

$$A = (1, 2, 3, 4, 5)$$

Then,

$$n(A) = 5$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{5}{6}$$

Question 4

A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(i) an ace

(ii) black card

Solution

Ans: Number of points in the sample space = 52

$$\therefore n(S) = 52$$

Ans: Let A be the event of drawing an ace of spades.

$$A = \{1\}$$

Then,

$$n(A) = 1$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{52}$$

(c) Calculate the probability that the card is

Ans: Let A be the event of drawing an ace. There are four aces.

Then,

$$n(A) = 4$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{4}{52}$$

$$P(A) = \frac{1}{13}$$

Ans: Let C be the event of drawing a black card. There are 26 black cards.

Then,

$$n(A) = 26$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{26}{52}$$

$$P(A) = \frac{1}{2}$$

Question 5

A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is

(i) 3

(ii) 12

Solution

Ans: As die and a coin marked 1 is tossed therefore, the sample space of possible outcomes will be,

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

Therefore,

$$n(S) = 12$$

Let A be the event having sum of numbers as 3.

$$A = \{(1, 2)\}$$

$$n(A) = 1$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{12}$$

Ans: Let A be the event having sum of number as 12.

$$A = \{(6, 6)\}$$

$$n(A) = 1$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{12}$$

Question 6

There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Solution

Ans:

$$\text{Total members in the council} = 4 + 6$$

$$\text{Total members in the council} = 10$$

Therefore,

$$n(S) = 10$$

Number of women are 6

Let A be the event of selecting a woman

Therefore,

$$n(A) = 6$$

$$P(E) = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{6}{10}$$

$$P(A) = \frac{3}{5}$$

Question 7

A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.50 for each tail that turns up.

From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution

Ans: Here, coin is tossed four times so the possible sample space contains,

$$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, TTTH, TTHT, THTT, HTTT, TTTT\}$$

According to question, a person will win or lose money depending up on the face of the coin so,

(i) For 4 heads

$$= 1 + 1 + 1 + 1$$

$$= ₹ 4$$

Therefore, he wins ₹ 4

(ii) For 3 heads and 1 tail

$$= 1 + 1 + 1 - 1.50$$

Therefore, he wins ₹ 1.50

(iii) For 2 heads and 2 tails = $1 + 1 - 1.50 - 1.50$

$$= 2 - 3$$

$$= - ₹ 1$$

Therefore, he will be losing ₹ 1

(iv) For 1 head and 3 tails = $1 - 1.50 - 1.50 - 1.50$

$$= 1 - 4.50$$

$$= - ₹ 3.50$$

Therefore, he will be losing Rs. 3.50

(v) For 4 tails = $- 1.50 - 1.50 - 1.50 - 1.50$

$$= - ₹ 6$$

Therefore, he will be losing Rs. 6

So, the new sample space will be

$$S = \{4, 1.50, 1.50, 1.50, 1.50, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -3.50, -3.50, -3.50, -3.50, -6\}$$

Then,

$$n(S) = 16$$

P (winning ₹ 4)

$$= \frac{1}{16}$$

P (winning ₹ 1.50)

$$= \frac{4}{16}$$

P (winning ₹ 1.50)

$$= \frac{1}{4}$$

P (loosing ₹ 1)

$$= \frac{6}{16}$$

P (loosing ₹ 1)

$$= \frac{3}{8}$$

P (loosing ₹ 3.50)

$$= \frac{4}{16}$$

P (loosing ₹ 3.50)

$$= \frac{1}{4}$$

P (loosing ₹ 6)

$$= \frac{1}{16}$$

Question 8

Three coins are tossed once. Find the probability of getting

(i) 3 heads

(ii) 2 heads

(iii) at least 2 heads

(iv) at most 2 heads

(v) no head

(vi) 3 tails

(vii) Exactly two tails

(viii) no tail

(ix) at most two tails

Solution

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting 3 heads

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{8}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting 2 heads

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{8}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting at least 2 head

$$n(A) = 4$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{4}{8}$$

$$P(A) = \frac{1}{2}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting at most 2 heads

$$n(A) = 7$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{7}{8}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting no heads

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{8}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting 3 tails

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{8}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting exactly 2 tails

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{8}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting no tails

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{8}$$

Ans: Three coin is tossed so the possible sample space contains,

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

$$n(S) = 8$$

Let A be the event of getting at most 2 tails

$$n(A) = 7$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{7}{8}$$

Question 9

If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Solution

Ans: According to question, it is given that, $\frac{2}{11}$ is the probability of an event A,

i.e.

$$P(A) = \frac{2}{11}$$

Therefore,

$$P(\text{not } A) = 1 - \frac{2}{11}$$

$$P(\text{not } A) = \frac{9}{11}$$

Question 10

A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is

(i) a vowel

(ii) a consonant

Solution

Ans: Total letters in the given word is 13

$$n(S) = 13$$

Number of vowels in the given word is 6

Let A be the event of selecting a vowel

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{6}{13}$$

Ans: Total letters in the given word is 13

$$n(S) = 13$$

Number of consonants in the given word = 7

Let A be the event of selecting the consonant

$$n(A) = 7$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{7}{13}$$

Question 11

In a lottery, a person chooses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? (Hint: order of the numbers is not important.)

Solution

Ans: According to question.

Total numbers of numbers in the draw is 20

Numbers to be selected is 6

Therefore,

$$n(S) = {}^{20}C_6$$

$$n(S) = \frac{20!}{6!(20-6)!}$$

$$n(S) = 38760$$

Let A be the event that six numbers match with the six numbers already fixed by the lottery committee.

$$n(A) = {}^6C_6$$

$$n(A) = 1$$

Therefore, the probability of winning the prize,

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{38760}$$

Question 12

Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined

(i)

$$P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$$

(ii)

$$P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$$

Solution

Ans:

$$P(A \cap B) > P(A)$$

Therefore, the given probabilities are not consistently defined.

Ans: As we know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.5 + 0.4 - P(A \cap B)$$

$$0.8 - 0.9 = -P(A \cap B)$$

$$P(A \cap B) = 0.1$$

Therefore,

$$P(A \cap B) < P(A)$$

and

$$P(A \cap B) < P(B)$$

So, the given probabilities are consistently defined.

Question 13

Fill in the blanks in following table:

	P(A)	P(B)	$P(A \cap B)$	$P(A \cup B)$
(i)	1/3	1/5	1/15	
(ii)	0.35		0.25	0.6
(iii)	0.5	0.35		0.7

Solution

Ans:

(i) We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{3} + \frac{1}{5} + \frac{1}{15}$$

$$P(A \cup B) = \frac{7}{15}$$

Ans:

(ii) We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = 0.35 + P(B) - 0.25$$

$$P(B) = 0.6 - 0.35 + 0.25$$

$$P(B) = 0.5$$

Ans:

(iii) We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.5 + 0.35 - P(A \cap B)$$

$$P(A \cap B) = 0.5 + 0.35 - 0.7$$

$$P(A \cap B) = 0.15$$

Question 14Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ or } B)$, if A and B are mutually exclusive events.**Solution**

Ans:

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

If A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = \frac{3}{5} + \frac{1}{5}$$

$$P(A \cup B) = \frac{4}{5}$$

Question 15

If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find

(i) $P(E \text{ or } F)$

(ii) $P(\text{not } E \text{ and not } F)$

Solution

Ans: We have,

$$P(E) = \frac{1}{4}$$

,

$$P(F) = \frac{1}{2}$$

,

$$P(E \cap F) = \frac{1}{8}$$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$P(A \cup B) = \frac{5}{8}$$

Ans:

$$P(\text{not } E \text{ and not } F) = P(\overline{E} \cap \overline{F})$$

$$P(\text{not } E \text{ and not } F) = P(\overline{E \cup F})$$

$$P(\text{not } E \text{ and not } F) = 1 - P(E \cup F)$$

$$P(\text{not } E \text{ and not } F) = 1 - \frac{5}{8}$$

$$P(\text{not } E \text{ and not } F) = \frac{3}{8}$$

Question 16

Events E and F are such that $P(\text{not } E \text{ or not } F) = 0.25$, State whether E and F are mutually exclusive.

Solution

Ans: We have,

$$P(\text{not } E \text{ or not } F) = 0.25$$

$$P(\overline{E} \cup \overline{F}) = 0.25$$

$$P(\overline{E \cap F}) = 0.25$$

$$1 - P(E \cap F) = 0.25$$

$$P(E \cap F) = 1 - 0.25$$

$$P(E \cap F) = 0.75$$

$$P(E \cap F) \neq 0$$

Therefore, E and F are not mutually exclusive event.

Question 17

A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine

(i) $P(\text{not } A)$

(ii) $P(\text{not } B)$

(iii) $P(A \text{ or } B)$

Solution

Ans: We know that,

$$P(\text{not } A) = 1 - P(A)$$

$$P(\text{not } A) = 1 - 0.42$$

$$P(\text{not } A) = 0.58$$

Ans: We know that,

$$P(\text{not } B) = 1 - P(B)$$

$$P(\text{not } B) = 1 - 0.48$$

$$P(\text{not } B) = 0.52$$

Ans: We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.42 + 0.48 - 0.16$$

$$P(A \cup B) = 0.74$$

Question 18

In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Solution

Ans: Let A be the event that the student is studying mathematics and B be the event that the student is studying biology,

Therefore,

$$P(A) = \frac{40}{100}$$

$$P(A) = \frac{2}{5}$$

And,

$$P(B) = \frac{30}{100}$$

$$P(B) = \frac{3}{10}$$

So, student studying both subjects,

$$P(A \cap B) = \frac{10}{100}$$

$$P(A \cap B) = \frac{1}{10}$$

Therefore, probability of studying mathematics or biology is given as,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{2}{5} + \frac{3}{10} - \frac{1}{10}$$

$$P(A \cup B) = \frac{6}{10}$$

$$P(A \cup B) = \frac{3}{5}$$

Therefore,

$$\frac{3}{5}$$

is the probability that student will studying mathematics or biology.

Question 19

In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Solution

Ans: Let probability of a randomly chosen student passing the first examination is 0.8 be

$$P(A)$$

.

And also probability of passing the second examination is 0.7 be

$$P(B)$$

Therefore,

$$P(A \cup B)$$

is probability of passing at least one of the examination

So, We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.95 = 0.8 + 0.7 - P(A \cap B)$$

$$P(A \cap B) = 0.8 + 0.7 - 0.95$$

$$P(A \cap B) = 0.55$$

therefor, 0.55 is the probability that student will pass both the examinations.

Question 20

The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Solution

Ans: Let probability of passing the English examination is 0.75 be

$$P(A)$$

.

And also assume the probability of passing the Hindi examination is be

$$P(B)$$

.

We hve given,

$$P(A) = 0.75, P(A \cap B) = 0.5, P(A' \cap B') = 0.1$$

We know that,

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$P(A \cup B) = 1 - P(A' \cap B')$$

$$P(A \cup B) = 1 - 0.1$$

$$P(A \cup B) = 0.9$$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.9 = 0.75 + P(B) - 0.5$$

$$P(B) = 0.9 + 0.5 - 0.75$$

$$P(B) = 0.65$$

Therefore, the probability of passing the hindi examination is 0.65.

Question 21

In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

- (i) The student opted for NCC or NSS.
- (ii) The student has opted neither NCC nor NSS.
- (iii) The student has opted NSS but not NCC.

Solution

Ans: The total number of students in class = 60

Therefore, sample space consist of

$$n(S) = 60$$

Let the students opted for NCC be 'A'

And the students opted for NSS be 'B'

$$n(A) = 30, n(B) = 32, n(A \cap B) = 24$$

Therefore,

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{30}{60}$$

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{32}{60}$$

$$P(B) = \frac{8}{15}$$

And,

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$P(A \cap B) = \frac{24}{60}$$

$$P(A \cap B) = \frac{2}{5}$$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{2} + \frac{8}{15} - \frac{2}{5}$$

$$P(A \cup B) = \frac{19}{30}$$

Ans:

$$P(\text{not } A \text{ and not } B) = P(A' \cap B')$$

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$P(A' \cap B') = 1 - \frac{19}{30}$$

$$P(A' \cap B') = \frac{11}{30}$$

Ans: P(student opted NSS but not NCC)

$$n(B - A) = n(B) - n(A \cap B)$$

$$n(B - A) = 32 - 24$$

$$n(B - A) = 8$$

The probability that the selected student has opted for NSS and not NCC is

$$P(B - A) = \frac{n(B - A)}{n(S)}$$

$$P(B - A) = \frac{8}{60}$$

$$P(B - A) = \frac{2}{15}$$

Miscellaneous Exercise

Question 1

A box contains

10

red marbles,

20

blue marbles and

$$30$$

green marbles.

$$5$$

marbles are drawn from the box, what is the probability that:

(i) All will be blue?

(ii) at least one will be green?

Solution

Ans. Total number of marbles

$$= 60$$

The number of ways of drawing

$$5$$

marbles from

$$60$$

marbles is

$${}^{60}C_5$$

.

All the marbles drawn will be blue if we draw

$$5$$

marbles out of the

$$20$$

blue marbles.

$$5$$

blue marbles can be drawn out of the

$$20$$

blue marbles in

$${}^{20}C_5$$

ways.

Probability that all marbles will be blue is

$$\frac{{}^{20}C_5}{{}^{60}C_5}$$

.

Ans. Total number of marbles

$$= 60$$

Number of ways of drawing

$$5$$

marbles from

$$60$$

marbles is

$${}^{60}C_5$$

.

The number of ways in which the drawn marbles is not green, that is, when red or blue marbles are drawn, is

$${}^{20+10}C_5 = {}^{30}C_5$$

.

So, probability that no marble is green is

$$\frac{{}^{30}C_5}{{}^{60}C_5}$$

.

Therefore, the probability that at least one marble will be green is

$$1 - \frac{{}^{30}C_5}{{}^{60}C_5}$$

.

Question 2

$$4$$

cards are drawn from a well-shuffled deck of

$$52$$

cards. What is the probability of obtaining three diamonds and one spade?

Solution

Ans. Number of ways of drawing

$$4$$

cards from a pack of

$$52$$

cards are

$${}^{52}C_4$$

.

In a deck, there are

$$13$$

diamonds and

$$13$$

spades.

So, the number of ways of drawing three diamonds and one spade is

$${}^{13}C_3 \times {}^{13}C_1$$

.

Hence, the probability of drawing three diamonds and one spade is

$$\frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$$

.

Question 3

A die has two faces each with the number ‘

$$1$$

’, three faces each with number ‘

$$2$$

’ and one face with number ‘

$$3$$

’. If die is rolled once, determine:

(i)

$$P(2)$$

(ii) $P(1 \text{ or } 3)$

(iii) $P(\text{not } 3)$

Solution

Ans. Total number of faces of the die is

$$6$$

.

The number of faces with the number ‘

$$2$$

' is

3

.

Therefore, probability of obtaining the number

2

is

$$P(2) = \frac{n(2)}{n(S)}$$

.

$$\Rightarrow P(2) = \frac{3}{6}$$

$$\Rightarrow P(2) = \frac{1}{2}$$

Ans. Total number of faces of the die is

6

.

Number of faces with the number '1

1

' is

2

and with the number '3

3

' is

1

.

Therefore, probability of obtaining the number

2

or

3

is

$$P(1 \cup 3) = \frac{n(1 \cup 3)}{n(S)}$$

.

$$\Rightarrow P(1 \cup 3) = \frac{1+2}{6}$$

$$\Rightarrow P(1 \cup 3) = \frac{1}{2}$$

Ans. Total number of faces of the die is

$$6$$

.

Number of faces with the number ‘

$$3$$

’ is

$$1$$

.

Therefore, probability of obtaining the number

$$3$$

is

$$P(3) = \frac{1}{6}$$

.

$$\Rightarrow P(3') = 1 - P(3)$$

$$\Rightarrow P(3') = 1 - \frac{1}{6}$$

$$\Rightarrow P(3') = \frac{5}{6}$$

Question 4

In a certain lottery,

$$10,000$$

tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy:

(i) One ticket?

(ii) two tickets?

(iii) ten tickets?

Solution

Ans. Total number of tickets sold is

$$10,000$$

.

Number of prizes awarded is

$$10$$

.

If A is the event we buy

$$1$$

ticket, the probability of winning will be

$$P(A) = \frac{n(A)}{n(S)}$$

.

$$\Rightarrow P(A) = \frac{10}{10,000}$$

$$\Rightarrow P(A) = \frac{1}{1,000}$$

So, the probability of not getting a prize is

$$P(A') = 1 - P(A)$$

.

$$\Rightarrow P(A') = 1 - \frac{1}{1,000}$$

$$\Rightarrow P(A') = \frac{999}{1,000}$$

Ans. Total number of tickets sold is

$$10,000$$

.

Number of prizes awarded is

$$10$$

.

Number of tickets on which a prize is not awarded is

$$10,000 - 10 = 9,990$$

.

If B is the event we buy

$$2$$

tickets, the probability of not winning will be

$$P(B) = \frac{{}^{9,990}C_2}{{}^{10,000}C_2}$$

.

Ans. Total number of tickets sold is

$$10,000$$

.

Number of prizes awarded is

$$10$$

.

Number of tickets on which a prize is not awarded is

$$10,000 - 10 = 9,990$$

.

If C is the event we buy

$$10$$

tickets, the probability of not winning will be

$$P(C) = \frac{{}^{9,990}C_{10}}{{}^{10,000}C_{10}}$$

.

Question 5

Out of

$$100$$

students, two sections of

$$40$$

and

$$60$$

are formed. If you and your friend are among the

$$100$$

students, what is the probability that

(i) You both enter the same section?

(ii) You both enter different sections?

Solution

Ans. You and your friend are among the

$$100$$

students.

Total number of ways of selecting

$$2$$

students out of

$$100$$

students is ${}^{100}C_2$.

If both of you enter the same section, you are either among the section having

$$40$$

students or the one having

$$60$$

students.

The number of ways in which you both can be in the same section is

$${}^{40}C_2 + {}^{60}C_2$$

.

The probability that you both can be in the same section is

$$\begin{aligned} & \frac{{}^{40}C_2 + {}^{60}C_2}{{}^{100}C_2} \\ & \Rightarrow \frac{\frac{40!}{2!38!} + \frac{60!}{2!58!}}{\frac{100!}{2!98!}} \\ & \Rightarrow \frac{(40 \times 39) + (60 \times 59)}{(100 \times 99)} \\ & \Rightarrow \frac{17}{33} \end{aligned}$$

Ans. The probability that you both can be in the same section is

$$\frac{17}{33}$$

.

The probability that you both are not in the same section will be

$$1 - \frac{17}{33}$$

$$\Rightarrow \frac{16}{33}$$

Question 6

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution

Ans. Let

$$L_1, L_2, L_3$$

be the three letters and

$$E_1, E_2, E_3$$

be their corresponding envelopes.

There are

$$6$$

ways of inserting

$$3$$

letters in

$$3$$

envelopes. These are as follows:

$$\begin{aligned} &L_1E_1, L_2E_3, L_3E_2 \\ &L_2E_2, L_1E_3, L_3E_1 \\ &L_3E_3, L_1E_2, L_2E_1 \\ &L_1E_1, L_2E_2, L_3E_3 \\ &L_1E_2, L_2E_3, L_3E_1 \\ &L_1E_3, L_2E_1, L_3E_2 \end{aligned}$$

We observe that there are

$$4$$

ways in which at least one letter is inserted in its proper envelope.

Hence, the required probability is

$$\frac{4}{6} = \frac{1}{2}$$

Question 7

A and B are two events such that

$$P(A) = 0.54$$

,

$$P(B) = 0.69$$

and

$$P(A \cap B) = 0.35$$

. Find:

(i)

$$P(A \cup B)$$

(ii)

$$P(A' \cap B')$$

(iii)

$$P(A \cap B')$$

(iv)

$$P(B \cap A')$$

Solution

Ans. It is given to us that

$$P(A) = 0.54$$

,

$$P(B) = 0.69$$

,

$$P(A \cap B) = 0.35$$

.

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.54 + 0.69 - 0.35$$

$$\Rightarrow P(A \cup B) = 0.88$$

Ans. It is given to us that

$$P(A) = 0.54$$

,

$$P(B) = 0.69$$

$$P(A \cap B) = 0.35$$

We know that

$$\begin{aligned} P(A' \cap B') &= P(A \cup B)' \\ \Rightarrow P(A' \cap B') &= 1 - P(A \cup B) \\ \Rightarrow P(A' \cap B') &= 1 - 0.88 \\ \Rightarrow P(A' \cap B') &= 0.12 \end{aligned}$$

Ans. It is given to us that

$$P(A) = 0.54$$

$$P(B) = 0.69$$

$$P(A \cap B) = 0.35$$

We know that

$$\begin{aligned} P(A \cap B') &= P(A) - P(A \cap B) \\ \Rightarrow P(A \cap B') &= 0.54 - 0.35 \\ \Rightarrow P(A \cap B') &= 0.19 \end{aligned}$$

Ans. It is given to us that

$$P(A) = 0.54$$

$$P(B) = 0.69$$

$$P(A \cap B) = 0.35$$

We know that

$$\begin{aligned} P(B \cap A') &= P(B) - P(A \cap B) \\ \Rightarrow P(B \cap A') &= 0.69 - 0.35 \end{aligned}$$

$$\Rightarrow P(B \cap A') = 0.34$$

Question 8

From the employees of a company,

5

persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No	Name	Sex	Age in years
1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alis	F	28
5	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over

35

years?

Solution

Ans. Let E be the event that the spokesperson will be a male and F be the event that the spokesperson will be over

35

years of age.

Subsequently,

$$P(E) = \frac{3}{5}$$

and

$$P(F) = \frac{2}{5}$$

.

Since there is only one male who is above

35

years of age,

$$P(E \cap F) = \frac{1}{5}$$

.

We know that

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\Rightarrow P(E \cup F) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$$

$$\Rightarrow P(E \cup F) = \frac{4}{5}$$

Hence, the probability that the spokesperson will either be a male or over

$$\frac{35}{100}$$

years of age is

$$\frac{4}{5}$$

.

Question 9

If four digit numbers greater than 5,000 are randomly formed from the digits

$$0$$

,

$$1$$

,

$$3$$

,

$$5$$

, and

$$7$$

, what is the probability of forming a number divisible by

$$5$$

when:

(i) the digits are repeated?

(ii) the repetition of digits is not allowed?

Solution

Ans. Since four-digit numbers greater than

$$5,000$$

are formed, the thousand's digit is either

$$5$$

or

$$7$$

. The remaining three digits can be filled by any of the given digits with repetition.

So, the total number of ways of forming four-digit numbers are

$$2 \times 5 \times 5 \times 5$$

, and subtract

$$1$$

from the obtained answer, as we cannot count

$$5,000$$

in the numbers.

$$\Rightarrow 250 - 1$$

$$\Rightarrow 249$$

ways

A number is divisible by

$$5$$

if the digit at its unit's place is either

$$0$$

or

$$5$$

.

Total number of four-digit numbers greater than

$$5,000$$

that are divisible by

$$5$$

are

$$(2 \times 5 \times 5 \times 2) - 1$$

.

$$\Rightarrow 100 - 1$$

$$\Rightarrow 99$$

Therefore, the probability of forming a number divisible by

$$5$$

when the digits are repeated is

$$\frac{99}{249} = \frac{33}{83}$$

Ans. Since four-digit numbers greater than

$$5,000$$

are formed, the thousand's digit is either

$$5$$

or

$$7$$

. The remaining three digits can be filled by any of the given digits without repetition.

So, the total number of ways of forming four-digit numbers is

$$2 \times 4 \times 3 \times 2 = 48$$

.

A number is divisible by

$$5$$

if the digit at its unit's place is either

$$0$$

or

$$5$$

.

When the digit at the thousands place is

$$5$$

, the unit place can be filled only with

$$0$$

, so the number of possible ways is

$$1 \times 3 \times 2 \times 1 = 6$$

.

When the digit at the thousands place is

$$7$$

, the units place can be filled only with

$$0$$

or

$$5$$

, so the number of possible ways are

$$1 \times 3 \times 2 \times 2 = 12$$

.

Total number of four-digit numbers greater than

$$5,000$$

that are divisible by

$$5$$

are

$$12 + 6 = 18$$

.

Therefore, the probability of forming a number divisible by

$$5$$

when the digits are repeated is

$$\frac{18}{48} = \frac{3}{8}$$

.

Question 10

The number lock of a suitcase has

$$4$$

wheels, each labelled with ten digits i.e., from

$$0$$

to

$$9$$

. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Solution

Ans. The number lock of a suitcase has

$$4$$

wheels, each of which is labelled with ten digits, that is, from

$$0$$

to

$$9$$

.

Number of ways of selecting

$$4$$

different digits out of

$$10$$

digits is

$${}^{10}C_4$$

.

Further, each combination of

$$4$$

different digits can be arranged in

$$4$$

ways.

Number of four digits with no repetitions

$$= {}^{10}C_4 \times 4!$$

$$\Rightarrow \frac{10!}{4!6!} \times 4!$$

$$\Rightarrow 10 \times 9 \times 8 \times 7$$

$$\Rightarrow 5040$$

There is only one number that can open the suitcase.

Thus, the required probability is

$$\frac{1}{5040}$$

.