

Chapter 13: Statistics

NCERT · Class 11 · Maths

Exercise 13.1

Question 1

Calculate the mean deviation about mean for the numbers

4, 7, 8, 9, 10, 12, 13, 17.

Solution

Ans.

We are provided the numbers

4, 7, 8, 9, 10, 12, 13, 17.

Therefore, mean of the numbers,

$$\begin{aligned}\bar{x} &= \frac{4 + 7 + 8 + 9 + 10 + 12 + 13 + 17}{8} \\ &= \frac{80}{8} \\ &= 10\end{aligned}$$

The deviations for the given numbers with the mean $\bar{x} = 10$,

that is $x_i - \bar{x}$, for $i = 1, 2, \dots, 8$ are

−6, −3, −2, −1, 0, 2, 3, 7.

Also, the absolute values of the obtained deviations, that is $|x_i - \bar{x}|$ are

6, 3, 2, 1, 0, 2, 3, 7.

Thus, the mean deviation about the mean $\bar{x}$ for the given numbers is

$$\begin{aligned} M.D. (\bar{x}) &= \frac{\sum_{i=1}^8 |x_i - \overline{line}x|}{8} \\ &= \frac{6 + 3 + 2 + 1 + 0 + 2 + 3 + 7}{8} \\ &= \frac{24}{8} \\ &= 3 \end{aligned}$$

Question 2

Calculate the mean deviation about the mean for the following numbers.

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution

Ans.

We are provided that numbers

38, 70, 48, 40, 42, 55, 63, 46, 54, 44.

Therefore, the mean of the numbers,

$$\begin{aligned} \bar{x} &= \frac{38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44}{10} \\ &= \frac{500}{10} \\ &= 50 \end{aligned}$$

Now, the deviations for the given numbers with the mean $\bar{x} = 50$, that is

$x_i - \bar{x}$, for $i = 1, 2, 3, \dots, 10$ are

$-12, 20, -2, -10, -8, 5, 13, -4, 4, -6$.

So, the absolute values of the obtained deviations for the given numbers, that is $|x_i - \bar{x}|$ are

$12, 20, 2, 10, 8, 5, 13, 4, 4, 6$.

Thus, the mean deviation about the mean $\bar{x}$ for the given numbers is

$$\begin{aligned} M.D. (\bar{x}) &= \frac{\sum_{i=1}^{10} |x_i - \bar{x}|}{10} \\ &= \frac{12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6}{10} \\ &= \frac{84}{10} \\ &= 8.4 \end{aligned}$$

Question 3

Calculate the mean deviation about the median for the following numbers.

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17.

Solution

Ans.

We are provided the numbers

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17.

There is total 12 numbers, and it is even.

Now, arranging the numbers in the ascending order, gives

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18.

It is known that,

$$\text{Median, } M = \frac{\left(\frac{n}{2}\right)^{th} \text{ number} + \left(\frac{n}{2} + 1\right)^{th} \text{ number}}{2}, \text{ when } n \text{ is even.}$$

Therefore, the median

$$\begin{aligned}
 M &= \frac{\left(\frac{12}{2}\right)^{th} \text{ number} + \left(\frac{12}{2} + 1\right)^{th} \text{ number}}{2} \\
 &= \frac{6^{th} \text{ number} + 7^{th} \text{ number}}{2} \\
 &= \frac{13 + 14}{2} \\
 &= \frac{27}{2} \\
 &= 13.5
 \end{aligned}$$

Now, the deviations about the median M for the given numbers, that is $x_i - M$, for $i = 1, 2, 3, \dots, 12$ are

$$-3.5, -2.5, -2.5, -1.5, -0.5, -0.5, 0.5, 2.5, 3.5, 3.5, 3.5, 4.5$$

So, the absolute values of the resulted deviations, that is $|x_i - M|$, for $i = 1, 2, 3, \dots, 12$ are $3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5$.

Thus, the mean deviation about the median M for the given numbers,

$$\begin{aligned}
 M.D.(M) &= \frac{\sum_{i=1}^{12} |x_i - M|}{12} \\
 &= \frac{3.5 + 2.5 + 2.5 + 1.5 + 0.5 + 0.5 + 0.5 + 2.5 + 2.5 + 3.5 + 3.5 + 4.5}{12} \\
 &= \frac{28}{12} \\
 &= 2.33
 \end{aligned}$$

Question 4

Calculate the mean deviation about the median for the following numbers.

36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

Solution

Ans.

We are provided the numbers

36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

There is total 10 numbers, and it is even.

Now, arranging the numbers in the ascending order, gives

36, 42, 45, 46, 46, 49, 51, 53, 60, 72.

It is known that,

$$\text{Median, } M = \frac{\left(\frac{n}{2}\right)^{th} \text{ number} + \left(\frac{n}{2} + 1\right)^{th} \text{ number}}{2}, \text{ when } n \text{ is even.}$$

Therefore, the median

$$\begin{aligned} M &= \frac{\left(\frac{10}{2}\right)^{th} \text{ number} + \left(\frac{10}{2} + 1\right)^{th} \text{ number}}{2} \\ &= \frac{5^{th} \text{ number} + 6^{th} \text{ number}}{2} \\ &= \frac{46 + 49}{2} \\ &= \frac{95}{2} \\ &= 47.5 \end{aligned}$$

Now, the deviations about the median M for the given numbers, that is $x_i - M$, for $i = 1, 2, 3, \dots, 10$ are

-11.5, -5.5, -2.5, -1.5, -1.5, 1.5, 3.5, 5.5, 12.5, 24.5

So, the absolute values of the resulted deviations, that is $|x_i - M|$, for $i = 1, 2, 3, \dots, 10$ are

11.5, 5.5, 2.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5.

Thus, the mean deviation about the median M for the given numbers,

$$\begin{aligned}
 M.D.(M) &= \frac{\sum_{i=1}^{10} |x_i - M|}{10} \\
 &= \frac{11.5 + 5.5 + 2.5 + 1.5 + 1.5 + 1.5 + 3.5 + 5.5 + 12.5 + 24.5}{10} \\
 &= \frac{70}{10} \\
 &= 7.
 \end{aligned}$$

Question 5

Use the following data to calculate the mean deviation about the mean.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Solution

Ans.

Consider the following table of data.

x_i	f_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18

25	5	125	11	55
Total	$\sum f_i = 25$	$\sum f_i x_i = 350$		$\sum f_i x_i - \bar{x} = 158$

Thus, from the above table we have,

$$N = \sum_{i=1}^5 f_i = 25,$$

$$\sum_{i=1}^5 f_i x_i = 350$$

Therefore, the mean,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^5 f_i x_i$$

$$= \frac{1}{25} \times 350$$

$$= 14$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^5 f_i |x_i - \bar{x}|$$

$$= \frac{1}{25} \times 158$$

$$= 6.32$$

Question 6

Use the following data to calculate the mean deviation about the mean.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution

Ans.

Consider the following table of data.

x_i	f_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
Total	$\sum f_i = 80$	$\sum f_i x_i = 4000$		$\sum f_i x_i - \bar{x} = 1280$

Thus, from the above table we have,

$$N = \sum_{i=1}^5 f_i = 80,$$

$$\sum_{i=1}^5 f_i x_i = 4000$$

Therefore, the mean,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^5 f_i x_i$$

$$= \frac{1}{80} \times 4000$$

$$= 50$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^5 f_i |x_i - \bar{x}|$$

$$= \frac{1}{80} \times 1280$$

$$= 16$$

.

Question 7

Use the following data to calculate the mean deviation about the median.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution

Ans.

Notice that the given data is already in ascending order.

Consider the following table that shows the cumulative frequencies of the provided data.

x_i	f_i	$c.f.$

5	8	8
7	6	14
9	2	16
10	2	18
12	2	20
15	6	26

Thus, from the above table we have,

$N = 26$, that is an even number.

Therefore, the median can be obtained by calculating the mean of the 13^{th} and 14^{th} observations and both the observations belongs to the cumulative frequency 14.

Therefore,

Median,

$$M = \frac{13^{th} \text{ observation} + 14^{th} \text{ observation}}{2}$$

$$= \frac{7 + 7}{2}$$

$$= 7$$

Now, the following table shows the absolute values of the deviations about the median, that is $|x_i - M|$.

$ x_i - M $	2	0	2	3	5	8	Total
-------------	---	---	---	---	---	---	-------

f_i	8	6	2	2	2	6	$\sum f_i = 26$
$f_i x_i - M $	16	0	4	6	10	48	$\sum f_i x_i - M = 84$

Thus, from the above table, we have

$$\sum_{i=1}^6 f_i = 26 \text{ and } \sum_{i=1}^6 f_i |x_i - M| = 84.$$

Hence, the required mean deviation about the median,

$$M.D(M) = \frac{1}{N} \sum_{i=1}^6 f_i |x_i - M|$$

$$= \frac{1}{26} \times 84$$

$$= 3.23$$

Question 8

Use the following data to calculate the mean deviation about the median.

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Solution

Ans.

Notice that the given data is already in ascending order.

Consider the following table that shows the cumulative frequencies of the provided data.

x_i	f_i	$c. f.$
15	3	3
21	5	8
27	6	14
30	7	21
35	8	29

Thus, from the above table we have,

$N = 29$, that is an odd number.

Therefore, the median will be the $\frac{29 + 1}{2} = \frac{30}{2} = 15^{th}$ observation.

Notice that, the 15^{th} observation belongs to the cumulative frequency 21. So, the median is the corresponding observation, that is 30.

Now, the following table shows the absolute values of the deviations about the median, that is $|x_i - M|$.

$ x_i - M $	15	9	3	0	5	Total
f_i	3	5	6	7	8	$\sum f_i = 29$
$f_i x_i - M $	45	45	18	0	40	$\sum f_i x_i - M = 148$

Thus, from the above table, we have

$$\sum_{i=1}^5 f_i = 29 \text{ and } \sum_{i=1}^5 f_i |x_i - M| = 148.$$

Hence, the required mean deviation about the median,

$$\begin{aligned} M.D(M) &= \frac{1}{N} \sum_{i=1}^5 f_i |x_i - M| \\ &= \frac{1}{29} \times 148 \\ &= 5.1 \end{aligned}$$

Question 9

Use the following data to calculate the mean deviation about the mean.

Income/ day	Number of persons
0 – 100	4
100 – 200	8
200 – 300	9
300 – 400	10
400 – 500	7
500 – 600	5
600 – 700	4
700 – 800	3

Solution

Ans.

Consider the following table of data.

Income/day	Number of persons f_i	Mid-point x_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972
300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1168
700 – 800	3	750	2250	392	1176
Total	$\sum f_i = 50$		$\sum f_i x_i = 17900$		7896

Thus, from the above table we have,

$$N = \sum_{i=1}^8 f_i = 50$$

,

$$\sum_{i=1}^8 f_i x_i = 17900$$

Therefore, the mean,

$$\begin{aligned}\bar{x} &= \frac{1}{N} \sum_{i=1}^8 f_i x_i \\ &= \frac{1}{50} \times 17900\end{aligned}$$

$$= 358$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^8 f_i |x_i - \bar{x}|$$

$$= \frac{1}{50} \times 7896$$

$$= 157.92$$

Question 10

Use the following data to calculate the mean deviation about the mean.

Height (cm)	Number of men
95 – 105	9
105 – 115	13
115 – 125	26
125 – 135	30
135 – 145	12
145 – 155	10

Solution

Ans.

Consider the following table of data.

Height (cm)	Number of men f_i	Mid-point x_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
95 – 105	9	100	900	25.3	227.7
105 – 115	13	110	1430	15.3	198.9
115 – 125	26	120	3120	5.3	137.8
125 – 135	30	130	3900	4.7	141
135 – 145	12	140	1680	14.7	176.4
145 – 155	10	150	1500	24.7	247
Total	$\sum f_i = 100$		$\sum f_i x_i = 12530$		1128.8

Thus, from the above table we have,

$$N = \sum_{i=1}^6 f_i = 100$$

,

$$\sum_{i=1}^6 f_i x_i = 12530$$

Therefore, the mean,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^6 f_i x_i$$

$$= \frac{1}{100} \times 12530$$

$$= 125.3$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^6 f_i |x_i - \bar{x}|$$

$$= \frac{1}{100} \times 1128.8$$

$$= 11.28$$

.

Question 11

Use the following data including ages of 100 men to find the mean deviation about the median.

Age	Number of men
16 – 20	5
21 – 25	6
26 – 30	12
31 – 35	14

36 – 40	26
41 – 45	12
46 – 50	16
51 – 55	9

Solution

Ans.

Observe that the provided data is not continuous. So, to convert these discrete data to continuous data, subtract 0.5 from the lower limit and add 0.5 to the upper limit of the ages.

So, consider the following table of data.

Age	Number of men f_i	Cumulative frequency (c.f.)	Mid-point x_i	$ x_i - Med. $	$f_i x_i - Med. $
15.5 – 20.5	5	5	18	20	100
20.5 – 25.5	6	11	23	15	90
25.5 – 30.5	12	23	28	10	120
30.5 – 35.5	14	37	33	5	70
35.5 – 40.5	26	63	38	0	0
40.5 – 45.5	12	75	43	5	60
45.5 – 50.5	16	91	48	10	160
50.5 – 55.5	9	100	53	15	135

Total	100				735
-------	-----	--	--	--	-----

Therefore, here $N = 100$ and it is an even number.

So, $\frac{N}{2} = \frac{100}{2} = 50$. That is, the cumulative frequency greater than 50 is 63. Thus, the median class = 35.5 – 40.5.

Now, we know that,

$$\text{Median, } M = l + \frac{\frac{N}{2} - C}{f} \times h$$

We have $l = 35.5$, $C = 37$, $f = 26$, $h = 5$. Also, $N = 100$.

Therefore,

$$M = 35.5 + \frac{50 - 37}{26} \times 5 = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38.$$

Hence, the mean deviation about the median for the given data is

$$M.D.(M) = \frac{1}{N} \sum_{i=1}^8 f_i |x_i - M|$$

$$= \frac{1}{100} \times 735$$

$$= 7.35$$

Exercise 13.2

Question 1

Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12.

Solution

Ans: Given data: 6, 7, 10, 12, 13, 4, 8, 12.

Mean can be given as:

$$\bar{x} = \frac{\sum_{i=1}^8 x_i}{n} = \frac{6 + 7 + 10 + 12 + 13 + 4 + 8 + 12}{8} = \frac{72}{8} = 9$$

.

The following table is obtained:

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	$6 - 9 = -3$	9
7	$7 - 9 = -2$	4
10	$10 - 9 = 1$	1
12	$12 - 9 = 3$	9
13	$13 - 9 = 4$	16
4	$4 - 9 = -5$	25
8	$8 - 9 = -1$	1
12	$12 - 9 = 3$	9
		74

Variance can be given as:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\sigma^2 = \frac{1}{8} \times 74$$

$$\sigma^2 = 9.25.$$

Therefore, mean is 9 and variance is 9.25.

Question 2

Find the mean and variance for the first n natural numbers.

Solution

Ans: The mean of first n natural numbers can be calculated as:

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

$$\therefore \text{Mean} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

Variance,

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n \left(x_i - \left(\frac{n+1}{2} \right) \right)^2$$

As we know, $(a - b)^2 = a^2 - 2ab + b^2$. Hence:

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n x_i^2 - \frac{1}{n} \sum_{i=1}^n 2 \left(\frac{n+1}{2} \right) x_i + \frac{1}{n} \sum_{i=1}^n \left(\frac{n+1}{2} \right)^2$$

$$\begin{aligned} \Rightarrow \frac{1}{n} \frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{2} \right) \left[\frac{n(n+1)}{2} \right] + \frac{(n+1)^2}{4n} \times n \\ \Rightarrow \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4} \end{aligned}$$

Taking LCM, we get:

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

Taking $(n+1)$ as common, we get:

$$\begin{aligned} \Rightarrow (n+1) \left[\frac{4n+2-3n-3}{12} \right] \\ \Rightarrow \frac{(n+1)(n-1)}{12} \end{aligned}$$

Now, we know, $(a+b)(a-b) = a^2 - b^2$. Hence,

$$\Rightarrow \frac{n^2-1}{12}$$

Therefore, mean is $\frac{n+1}{2}$ and variance is $\frac{n^2-1}{12}$.

Question 3

Find the mean and variance for the first 10 multiples of 3.

Solution

Ans: First 10 multiples of 3 can be written as: 3, 9, 12, 15, 18, 21, 24, 27, 30. Hence, $n = 10$.

Mean,

$$\bar{x} = \frac{\sum_{i=1}^8 x_i}{n} = \frac{3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30}{10} = \frac{165}{10} = 16.5$$

The following table is obtained:

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
-------	-------------------	---------------------

3	$3 - 16.5 = -13.5$	182.25
6	$6 - 16.5 = -10.5$	110.25
9	$9 - 16.5 = -7.5$	56.25
12	$12 - 16.5 = -4.5$	20.25
15	$15 - 16.5 = -1.5$	2.25
18	$18 - 16.5 = 1.5$	2.25
21	$21 - 16.5 = 4.5$	20.25
24	$24 - 16.5 = 7.5$	56.25
27	$27 - 16.5 = 10.5$	110.25
30	$30 - 16.5 = 13.5$	182.25
		742.5

Variance can be given as:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\sigma^2 = \frac{1}{10} \times 742.5$$

$$\sigma^2 = 74.25.$$

Therefore, mean is 16.5 and variance is 74.25.

Question 4

Find the mean and variance for the data:

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Solution

Ans: Calculating the given data we get the following table:

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	$6 - 19 = -13$	169	338
10	4	40	$10 - 19 = -9$	81	324
14	7	98	$14 - 19 = -5$	25	175
18	12	216	$18 - 19 = -1$	1	12
24	8	192	$24 - 19 = 5$	25	200
28	4	112	$28 - 19 = 9$	81	324
30	3	90	$30 - 19 = 11$	121	363
	40	760			1736

Here,

$$N = 40$$

,

$$\sum_{i=1}^7 f_i x_i = 760$$

$$\sum_{i=1}^7 f_i(x - \bar{x})^2 = 1736$$

Mean,

$$\bar{x} = \frac{\sum_{i=1}^7 f_i x_i}{\sum_{i=1}^7 N} = \frac{1}{40} \times 760 = 19$$

$$\text{Variance, } \sigma^2 = \frac{1}{N} \sum_{i=1}^7 f_i(x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$$

Therefore, mean is 19 and variance is 43.4.

Question 5

Find the mean and variance for the data:

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution

Ans: Calculating the given data we get the following table:

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	$92 - 100 = -8$	64	192
93	2	186	$93 - 100 = -7$	49	98
97	3	291	$97 - 100 = -3$	9	27

98	2	196	$98 - 100 = -2$	4	8
102	6	612	$102 - 100 = 2$	4	24
104	3	312	$104 - 100 = 4$	16	48
109	3	327	$109 - 100 = 9$	81	243
	22	2200			640

Here,

$$N = 22$$

,

$$\sum_{i=1}^7 f_i x_i = 2200$$

,

$$\sum_{i=1}^7 f_i (x_i - \bar{x})^2 = 640$$

Mean,

$$\bar{x} = \frac{\sum_{i=1}^7 f_i x_i}{N} = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance, } \sigma^2 = \frac{1}{N} \sum_{i=1}^7 f_i (x_i - \bar{x})^2 = \frac{1}{22} \times 640 = 29.09$$

Therefore, mean is 100 and variance is 29.09.

Question 6

Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution

Ans: Suppose mean $A = 54$ and here, $h = 1$.

Calculating the given data, we get the following table:

x_i	f_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	100	220		0	286

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^9 f_i y_i}{\sum_{i=1}^9 N} \times h$$

$$= 64 + \frac{0}{100} \times 1$$

$$= 64 + 0$$

$$= 64$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^9 f_i y_i^2 - \left(\sum_{i=1}^9 f_i y_i \right)^2 \right]$$

$$= \frac{(1)^2}{(100)^2} [100(286) - (0)^2]$$

$$= 2.86$$

Hence, standard deviation, $\sigma = \sqrt{2.86} = 1.69$

Therefore, mean is 64 and standard deviation is 1.69.

Question 7

Find the mean and variance for the following frequency distribution.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution

Ans: Suppose mean $A = 105$ and here, $h = 30$.

Calculating the given data, we get the following table:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
-------	-----------------	-----------------	---------------------------	---------	-----------	-------------

0-30	2	15	-3	9	-6	18
30-60	3	45	- 2	4	- 6	12
60-90	5	75	- 1	1	- 5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^7 f_i y_i}{\sum_{i=1}^7 N} \times h$$

$$= 105 + \frac{2}{30} \times 30$$

$$= 105 + 2$$

$$= 107$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^7 f_i y_i^2 - \left(\sum_{i=1}^7 f_i y_i \right)^2 \right]$$

$$= \frac{(30)^2}{(30)^2} [30(76) - (2)^2]$$

$$= 2280 - 4$$

$$= 2276$$

Therefore, mean is 107 and variance is 2276.

Question 8

Find the mean and variance for the following frequency distribution.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution

Ans: Suppose mean $A = 25$ and here, $h = 10$.

Calculating the given data, we get the following table:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-10	5	5	-2	4	-10	20
10-20	8	15	-1	1	-8	8
20-30	15	25	0	0	0	0
30-40	16	35	1	1	16	16
40-50	6	45	2	4	12	24
	50				10	68

Mean,

$$\begin{aligned}\bar{x} &= A + \frac{\sum_{i=1}^5 f_i y_i}{\sum_{i=1}^5 N} \times h \\ &= 25 + \frac{10}{50} \times 10\end{aligned}$$

$$= 25 + 2$$

$$= 27$$

$$\begin{aligned}\text{Variance, } \sigma^2 &= \frac{h^2}{N^2} \left[N \sum_{i=1}^5 f_i y_i^2 - \left(\sum_{i=1}^5 f_i y_i \right)^2 \right] \\ &= \frac{(10)^2}{(50)^2} [50(68) - (10)^2] \\ &= \frac{1}{25} [50 \times 68 - (10)^2] \\ &= \frac{3300}{25} \\ &= 132\end{aligned}$$

Therefore, mean is 27 and variance is 132.

Question 9

Find the mean, variance and standard deviation using short-cut method.

Height in cm	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	2	1	12	29	25	12	10	4	5

Solution

Ans: Suppose mean $A = 92.5$ and here, $h = 5$.

Calculating the given data, we get the following table:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	- 4	16	-12	48
75-80	4	77.5	- 3	9	- 12	36

80-85	7	82.5	- 2	4	- 14	28
85-90	7	87.5	- 1	1	- 7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^9 f_i y_i}{\sum_{i=1}^9 N} \times h$$

$$= 92.5 + \frac{6}{60} \times 5$$

$$= 92.5 + 0.5$$

$$= 93$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^9 f_i y_i^2 - \left(\sum_{i=1}^9 f_i y_i \right)^2 \right]$$

$$= \frac{(5)^2}{(60)^2} [60(254) - (6)^2]$$

$$= \frac{25}{3600} (15204)$$

$$= 105.58$$

Hence, standard deviation, $\sigma = \sqrt{105.58} = 10.27$

Therefore, mean is 93, variance is 105.58 and standard deviation is 10.27.

Question 10

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

(Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 – 48.5, 48.5 – 52.5 and then proceed.)

Solution

Ans: Suppose mean $A = 42.5$ and here, $h = 4$.

Calculating the given data, we get the following table:

Class Interval	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
32.5-36.5	15	34.5	-2	4	-30	60
36.5-40.5	17	38.5	-1	1	-17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	100				25	199

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^5 f_i y_i}{\sum_{i=1}^5 N} \times h$$

$$= 42.5 + \frac{25}{100} \times 4$$

$$= 42.5 + 1$$

$$= 43.5$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^5 f_i y_i^2 - \left(\sum_{i=1}^5 f_i y_i \right)^2 \right]$$

$$= \frac{(4)^2}{(100)^2} [100(19) - (25)^2]$$

$$= \frac{16}{10000} (19275)$$

$$= 30.84$$

Hence, standard deviation, $\sigma = \sqrt{30.84} = 5.55$

Therefore, mean is 43.5, variance is 30.84 and standard deviation is 5.55.

Miscellaneous Exercise

Question 1

The mean and variance of eight observations are 9 and 9.25 respectively. If six of the observations are

6, 7, 10, 12, 12

and

13

, find the remaining two observations.

Solution

Ans: Consider the remaining two observations to be x and y . Thus the eight observations are

6, 7, 10, 12, 12, 13, x , y

Therefore, from the mean of the data it can be obtained that,

$$\bar{x} = \frac{6 + 7 + 10 + 12 + 12 + 13 + x + y}{8} = 9$$

$$\Rightarrow 60 + x + y = 72$$

$$\Rightarrow x + y = 12$$

.....(i)

Again, from the variance of the data it can be obtained that,

$$\sigma^2 = \frac{\sum_{i=1}^8 (x_i - \bar{x})^2}{N} = 9.25$$

$$\therefore \frac{1}{8} [(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (4)^2 + x^2 + y^2 - (2)(9)(x + y) + (2(9)^2)] = 9.25$$

$$\Rightarrow \frac{1}{8} [9 + 4 + 9 + 16 + x^2 + y^2 - (18)(12) + 162] = 9.25$$

[By, using (i)]

$$\Rightarrow \frac{1}{8} [x^2 + y^2 - 6] = 9.25$$

$$\Rightarrow x^2 + y^2 = 80$$

.....(ii)

Observe that from equation (i), it can be obtained that,

$$x^2 + y^2 + 2xy = 144 \text{(iii)}$$

Also, from equations (ii) and (iii), it can be obtained that,

$$2xy = 64$$

.....(iv)

Now, on subtracting equation (iv) from equation (ii), it can be obtained that,

$$x^2 + y^2 - 2xy = 16$$

$$\Rightarrow x - y = \pm 4$$

.....(v)

It can be calculated from equations (i) and (v) that, $x = 8$ and $y = 4$, when

$$x - y = 4$$

and $x = 4$ and $y = 8$, when

$$x - y = -4$$

. Henceforth, the remaining two observations are 4 and 8.

Question 2

The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are

$$2, 4, 10, 12$$

and

$$14$$

. Find the remaining two observations.

Solution

Ans: Consider the remaining two observations to be x and y . Thus the eight observations are

$$2, 4, 10, 12, 14, x, y$$

.

Therefore, from the mean of the data it can be obtained that,

$$\bar{x} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$$

$$\Rightarrow 42 + x + y = 56$$

$$\Rightarrow x + y = 14$$

.....(i)

Again, from the variance of the data it can be obtained that,

$$\sigma^2 = \frac{\sum_{i=1}^7 (x_i - \bar{x})^2}{N} = 16$$

$$\therefore \frac{1}{7} \left[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - (2)(8)(x + y) + (2(8)^2) \right] = 16$$

$$\Rightarrow \frac{1}{7} [36 + 16 + 4 + 16 + x^2 + y^2 - (16)(14) + 128] = 16$$

[By, using (i)]

$$\Rightarrow \frac{1}{7} [x^2 + y^2 + 12] = 16$$

$$\Rightarrow x^2 + y^2 = 100$$

.....(ii)

Observe that from equation (i), it can be obtained that,

$$x^2 + y^2 + 2xy = 196 \text{(iii)}$$

Also, from equations (ii) and (iii), it can be obtained that,

$$2xy = 96$$

.....(iv)

Now, on subtracting equation (iv) from equation (ii), it can be obtained that,

$$x^2 + y^2 - 2xy = 4$$

$$\Rightarrow x - y = \pm 2$$

.....(v)

It can be calculated from equations (i) and (v) that, $x = 8$ and $y = 6$, when

$$x - y = 2$$

and $x = 6$ and $y = 8$, when

$$x - y = -2$$

. Henceforth, the remaining two observations are 6 and 8.

Question 3

The mean and standard deviation of six observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution

Ans: Assume the observations to be

$$x_1, x_2, x_3, x_4, x_5, x_6$$

.

Therefore, the mean of the data is,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8$$

$$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8$$

.....(i)

Observe that when each of the observation is multiplied with 3 and if we consider the resulting observations as y_i , then observe as shown below,

$$\therefore y_1 = 3x_1$$

$$\Rightarrow x_1 = \frac{1}{3}y_1, \forall i = 1, 2, 3, \dots, 6 \& i \in \mathbb{Z}^+$$

Now, the mean of the new data is,

$$\therefore \bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$\Rightarrow \bar{y} = \frac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$$

$$\Rightarrow \bar{y} = 3 \times 8$$

[By using (i)]

$$\Rightarrow \bar{y} = 24$$

Therefore, the standard deviation of the data is,

$$\sigma = \sqrt{\frac{\sum_{i=1}^6 (x_i - \bar{x})^2}{n}} = 4$$

$$\Rightarrow (4)^2 = \frac{\sum_{i=1}^6 (x_i - \bar{x})^2}{6}$$

$$\Rightarrow \sum_{i=1}^6 (x_i - \bar{x})^2 = 96$$

.....(ii)

Again, it can be observed from equations (i) and (ii) that

$$\bar{y} = 3\bar{x}$$

and

$$\bar{x} = \frac{\bar{y}}{3}$$

and hence on substituting the values of x_i and $\bar{x}$ in equation (ii) it can be clearly obtained as shown below,

$$\therefore \sum_{i=1}^6 \left(\frac{y_i}{3} - \frac{\bar{y}}{3} \right)^2 = 96$$

$$\Rightarrow \sum_{i=1}^6 (y_i - \bar{y})^2 = 864$$

Henceforth, the standard deviation of the new data can be calculated as shown below,

$$\therefore \sigma = \sqrt{\frac{864}{6}} = \sqrt{144} = 12$$

Question 4

Given that $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, \dots, x_n$. Prove that the mean and variance of observations $ax_1, ax_2, ax_3, \dots, ax_n$ are $a\bar{x}$ and $a^2\sigma^2$, respectively ($a \neq 0$).

Solution

Ans: Observe that the given observations are $x_1, x_2, \dots, x_n$ and the mean and variance of data is $\bar{x}$ and σ^2 respectively.

$$\therefore \sigma^2 = \frac{\sum_{i=1}^n y_i (x_i - \bar{x})^2}{n}$$

.....(i)

Observe that when each of the observation is multiplied with a and if we consider the resulting observations as y_i , then observe as shown below,

$$\therefore y_i = ax_i$$

$$\Rightarrow x_i = \frac{1}{a}y_i, \forall i = 1, 2, 3, \dots, n \& i \in \mathbb{Z}^+$$

Now, the mean of the new data is,

$$\therefore \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i = \frac{1}{n} \sum_{i=1}^n ax_i = \frac{a}{n} \sum_{i=1}^n x_i = a\bar{x} \quad \left[\because \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \right]$$

Again on substituting the values of x_i and $\bar{x}$ in equation (i) it can be clearly obtained as shown below,

$$\therefore \sigma^2 = \frac{\sum_{i=1}^n \left(\frac{y_i}{a} - \frac{\bar{y}}{a} \right)^2}{n}$$

$$\Rightarrow a^2 \sigma^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n}$$

Henceforth, it can be clearly proved that the mean and variance of the new data is $a\bar{x}$ and $a^2\sigma^2$, respectively.

Question 5

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted

(ii) If it is replaced by 12.

Solution

Ans: Observe that the incorrect number of observations, incorrect mean and the incorrect standard deviation are 20, 10 and 2, respectively. $x_1, x_2, \dots, x_n$ and the mean and variance of data is $\bar{x}$ and σ^2 respectively.

$$\therefore \bar{x} = \frac{\sum_{i=1}^{20} x_i}{n} = 10$$

$$\Rightarrow \frac{\sum_{i=1}^{20} x_i}{20} = 10$$

$$\Rightarrow \sum_{i=1}^{20} x_i = 200$$

Now, the correct mean is,

$$\therefore \bar{x} = \frac{\sum_{i=1}^{19} x_i}{n}$$

$$\Rightarrow \bar{x} = \frac{192}{19} = 10.1 \quad \left[\because \sum_{i=1}^{19} x_i = 192 \right]$$

Again observe as shown below,

$$\therefore \sigma = \sqrt{\frac{1}{20} \sum_{i=1}^{20} x_i^2 - (\bar{x})^2} = 2$$

$$\Rightarrow 4 = \frac{1}{20} \sum_{i=1}^{20} x_i^2 - (\bar{x})^2$$

$$\Rightarrow 2080 = \sum_{i=1}^{20} x_i^2$$

Therefore, the correct standard deviation of the data is,

$$\sum_{i=1}^{20} x_i^2 - (8)^2$$

$$\Rightarrow 2080 - 64$$

$$\Rightarrow 2016$$

$$\therefore \sigma = \sqrt{\frac{\sum_{i=1}^{19} x_i^2}{n} - (\bar{x})^2}$$

$$\Rightarrow \sigma = \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$\Rightarrow \sigma = \sqrt{1061.1 - 102.1} = \sqrt{4.09} = 2.02$$

Ans: Observe that the incorrect sum of observations is 200.

$$\therefore \sum_{i=1}^{20} x_i = 200 - 8 + 12 = 204$$

Now, the correct mean is,

$$\therefore \bar{x} = \frac{\sum_{i=1}^{20} x_i}{n}$$

$$\Rightarrow \bar{x} = \frac{204}{20} = 10.2 \quad \left[\because \sum_{i=1}^{20} x_i = 204 \right]$$

Again observe as shown below,

$$\therefore \sigma = \sqrt{\frac{1}{20} \sum_{i=1}^{20} x_i^2 - (\bar{x})^2} = 2$$

$$\Rightarrow 4 = \frac{1}{20} \sum_{i=1}^{20} x_i^2 - (\bar{x})^2$$

$$\Rightarrow 2080 = \sum_{i=1}^{20} x_i^2$$

Therefore, the correct standard deviation of the data is,

$$\sum_{i=1}^{20} x_i^2 - (8)^2 + (12)^2$$

$$\Rightarrow 2080 - 64 + 144$$

$$\Rightarrow 2160$$

$$\therefore \sigma = \sqrt{\frac{\sum_{i=1}^{20} x_i^2}{n} - (\bar{x})^2}$$

$$\Rightarrow \sigma = \sqrt{\frac{2160}{20} - (10.2)^2}$$

$$\Rightarrow \sigma = \sqrt{108 - 104.04} = \sqrt{3.96} = 1.98$$

Question 6

The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that 3 observations are incorrect which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Solution

Ans: Observe that the number of observations, incorrect mean and the incorrect standard deviation are 100, 20 and 3, respectively. $x_1, x_2, \dots, x_n$ and the mean and variance of data is $\bar{x}$ and σ^2 respectively.

$$\therefore \bar{x} = \frac{\sum_{i=1}^{100} x_i}{n} = 20$$

$$\Rightarrow \frac{\sum_{i=1}^{100} x_i}{100} = 20$$

$$\Rightarrow \sum_{i=1}^{100} x_i = 2000$$

Now, the correct mean is,

$$\therefore \bar{x} = \frac{\sum_{i=1}^{97} x_i}{n}$$

$$\Rightarrow \bar{x} = \frac{2000 - 60}{97} = 20 \quad \left[\because \sum_{i=1}^{97} x_i = 1940 \right]$$

Again observe as shown below,

$$\therefore \sigma = \sqrt{\frac{1}{100} \sum_{i=1}^{100} x_i^2 - (\bar{x})^2} = 3$$

$$\Rightarrow 9 = \frac{1}{100} \sum_{i=1}^{100} x_i^2 - (\bar{x})^2$$

$$\Rightarrow 40900 = \sum_{i=1}^{100} x_i^2$$

Therefore, the correct standard deviation of the data is,

$$\sum_{i=1}^{100} x_i^2 - (2)(21)^2 - (18)^2$$

$$\Rightarrow 40900 - 1206$$

$$\Rightarrow 39694$$

$$\therefore \sigma = \sqrt{\frac{\sum_{i=1}^{97} x_i^2}{n} - (\bar{x})^2}$$

$$\Rightarrow \sigma = \sqrt{\frac{39694}{97} - (20)^2}$$

$$\Rightarrow \sigma = \sqrt{409.22 - 400} = \sqrt{9.22} = 3.04$$

.