

Chapter 13: Limits and Derivatives Exercise 13.2

NCERT · Class 11 · Maths

Questions

Question 1

Find the derivative of $x^2 - 2$ at $x = 10$.

Solution

Let $f(x) = x^2 - 2$

From first principle

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Put $x = 10$, we get

$$f'(10) = \lim_{h \rightarrow 0} \frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(10+h)^2 - 2] - (10^2 - 2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{10^2 + 2 \times 10 \times h + h^2 - 2 - 10^2 + 2}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{20h + h^2}{h} \\ &= \lim_{h \rightarrow 0} (20 + h) \\ &= 20 + 0 \\ &= 20 \end{aligned}$$

Question 2

Find the derivative of x at $x = 1$.

Solution

Let $f(x) = x$

Then,

From first principle

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Let $f(x) = x$

From first principle

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(10)}{h}$$

Put $x = 1$, we get

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(1 + h) - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1 + h - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h}$$

$$= \lim_{h \rightarrow 0} 1$$

$$= 1$$

Question 3

Find the derivative of $99x$ at $x = 100$.

Solution

Let $f(x) = 99x$,

From the first principle,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Put $x = 100$, we get

$$f'(100) = \lim_{h \rightarrow 0} \frac{f(100+h) - f(100)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{99(100+h) - 99 \times 100}{h}$$

$$= \lim_{h \rightarrow 0} \frac{99 \times 100 + 99h - 99 \times 100}{h}$$

$$= \lim_{h \rightarrow 0} \frac{99 \times h}{h}$$

$$= \lim_{h \rightarrow 0} 99$$

$$= 99$$

Question 4

Find the derivative of the following functions from the first principle.

(i) $x^3 - 27$

(ii) $(x-1)(x-2)$

(iii) $1/x^2$

(iv) $x + 1/x - 1$

Solution

(i) Let $f(x) = x^3 - 27$

From the first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[(x+h)^3 - 27] - (x^3 - 27)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^3 + h^3 + 3x^2h + 3xh^2 - x^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h^3 + 3x^2h + 3xh^2}{h} \\
 &= \lim_{h \rightarrow 0} (h^2 + 3x^2 + 3xh) \\
 &= 0 + 3x^2 \\
 &= 3x^2
 \end{aligned}$$

(ii) Let $f(x) = (x-1)(x-2)$

From the first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h-1)(x+h-2) - (x-1)(x-2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x^2 + hx - 2x + hx + h^2 - 2h - x - h + 2) - (x^2 - 2x - x + 2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{hx + hx + h^2 - 2h - h}{h} \\
 &= \lim_{h \rightarrow 0} (h + 2x - 3)
 \end{aligned}$$

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$$= 0 + 2x - 3$$

$$= 2x - 3$$

(iii) Let $f(x) = 1/x^2$

From the first principle, we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{x^2 - x^2 - h^2 - 2hx}{x^2(x+h)^2} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-h^2 - 2hx}{x^2(x+h)^2} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{-h - 2x}{x^2(x+h)^2} \right] \\ &= (0 - 2x) / [x^2(x+0)^2] \\ &= (-2/x^3) \end{aligned}$$

(iv) Let $f(x) = x + 1/x - 1$

From the first principle, we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x-1)(x+h+1) - (x+1)(x+h-1)}{h(x-1)(x+h-1)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{(x^2 + hx + x - x - h - 1) - (x^2 + hx + x - x + h - 1)}{(x-1)(x+h-1)} \right] \\
 &= \lim_{h \rightarrow 0} \frac{-2h}{h(x-1)(x+h-1)} \\
 &= \lim_{h \rightarrow 0} \frac{-2}{(x-1)(x+h-1)} \\
 &= -\frac{2}{(x-1)(x-1)} \\
 &= -\frac{2}{(x-1)^2}
 \end{aligned}$$

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Question 5

For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$, prove that $f'(1) = 100 f'(0)$.

Solution

Given function is:

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$$

By differentiating both sides, we get

$$\frac{d}{dx}f(x) = \frac{d}{dx} \left[\frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1 \right]$$

$$= \frac{d}{dx} \left(\frac{x^{100}}{100} \right) + \frac{d}{dx} \left(\frac{x^{99}}{99} \right) + \dots + \frac{d}{dx} \left(\frac{x^2}{2} \right) + \frac{d}{dx}(x) + \frac{d}{dx}(1)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\therefore \frac{d}{dx}f(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \dots + \frac{2x}{2} + 1 + 0$$

$$f'(x) = x^{99} + x^{98} + \dots + x + 1$$

At $x = 0$, we get

$$f'(0) = 0 + 0 + \dots + 0 + 1$$

$$f'(0) = 1$$

At $x = 1$, we get

$$f'(1) = 1^{99} + 1^{98} + \dots + 1 + 1 = [1 + 1 + \dots + 1] \text{ 100 times} = 1 \times 100 = 100$$

Hence, $f'(1) = 100 f'(0)$

Question 6

Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + \dots + a^{n-1}x + a^n$ for some fixed real number a .

Solution

Given function is:

$$f(x) = x^n + ax^{n-1} + a^2x^{n-2} + \dots + a^{n-1}x + a^n$$

By differentiating both sides, we get

$$\begin{aligned} f'(x) &= \frac{d}{dx} (x^n + ax^{n-1} + a^2x^{n-2} + \dots + a^{n-1}x + a^n) \\ &= \frac{d}{dx}(x^n) + a \frac{d}{dx}(x^{n-1}) + a^2 \frac{d}{dx}(x^{n-2}) + \dots + a^{n-1} \frac{d}{dx}(x) + a^n \frac{d}{dx}(1) \end{aligned}$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \dots + a^{n-1} + a^n(0)$$

$$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \dots + a^{n-1}$$

Question 7

For some constants a and b, find the derivative of

(i) $(x - a)(x - b)$

(ii) $(ax^2 + b)^2$

(iii) $x - a / x - b$

Solution

(i) $(x - a)(x - b)$

$$\text{Let } f(x) = (x - a)(x - b)$$

$$f(x) = x^2 - (a + b)x + \underline{ab}$$

Now, by differentiating both sides, we get

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^2 - (a + b)x + ab) \\ &= \frac{d}{dx}(x^2) - (a + b)\frac{d}{dx}(x) + \frac{d}{dx}(ab) \end{aligned}$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\begin{aligned} f'(x) &= 2x - (a + b) + 0 \\ &= 2x - a - b \end{aligned}$$

$$(ii) (ax^2 + b)^2$$

$$\text{Let } f(x) = (ax^2 + b)^2$$

$$f(x) = a^2x^4 + 2abx^2 + b^2$$

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx}(a^2x^4 + 2abx^2 + b^2)$$

$$f'(x) = \frac{d}{dx}(x^4) + (2ab)\frac{d}{dx}(x^2) + \frac{d}{dx}(b^2)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$f'(x) = a^2 \times 4x^3 + 2ab \times 2x + 0$$

$$= 4a^2x^3 + 4abx$$

$$= 4ax(ax^2 + b)$$

$$(iii) \ x - a / x - b$$

$$\text{Let } f(x) = \frac{(x-a)}{(x-b)}$$

By differentiating both sides and using quotient rule, we get

$$f'(x) = \frac{d}{dx}\left(\frac{x-a}{x-b}\right)$$

$$f'(x) = \frac{(x-b)\frac{d}{dx}(x-a) - (x-a)\frac{d}{dx}(x-b)}{(x-b)^2}$$

$$= \frac{(x-b)(1) - (x-a)(1)}{(x-b)^2}$$

By further calculation, we get

$$= \frac{x-b-x+a}{(x-b)^2}$$

$$= \frac{a-b}{(x-b)^2}$$

Question 8

$$\frac{x^n - a^n}{x - a}$$

Find the derivative of $\frac{x^n - a^n}{x - a}$ for some constant a .

Solution

$$\text{Let } f(x) = \frac{x^n - a^n}{x - a}$$

By differentiating both sides and using quotient rule, we get

$$f'(x) = \frac{d}{dx} \left(\frac{x^n - a^n}{x - a} \right)$$

$$f'(x) = \frac{(x-a) \frac{d}{dx} (x^n - a^n) - (x^n - a^n) \frac{d}{dx} (x-a)}{(x-a)^2}$$

By further calculation, we get

$$= \frac{(x-a)(nx^{n-1} - 0) - (x^n - a^n)(1)}{(x-a)^2}$$

$$= \frac{nx^n - anx^{n-1} - x^n + a^n}{(x-a)^2}$$

Question 9

Find the derivative of

(i) $2x - 3 / 4$

$$(ii) (5x^3 + 3x - 1)(x - 1)$$

$$(iii) x^{-3}(5 + 3x)$$

$$(iv) x^5(3 - 6x^{-9})$$

$$(v) x^{-4}(3 - 4x^{-5})$$

$$(vi) (2/x + 1) - x^2/3x - 1$$

Solution

(i)

$$\text{Let } f(x) = 2x - 3/4$$

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx} \left(2x - \frac{3}{4} \right)$$

$$= 2 \frac{d}{dx}(x) - \frac{d}{dx} \left(\frac{3}{4} \right)$$

$$= 2 - 0$$

$$= 2$$

(ii)

$$\text{Let } f(x) = (5x^3 + 3x - 1)(x - 1)$$

By differentiating both sides and using the product rule, we get

$$f'(x) = (5x^3 + 3x - 1) \frac{d}{dx}(x - 1) + (x - 1) \frac{d}{dx}(5x^3 + 3x - 1)$$

$$= (5x^3 + 3x - 1) \times 1 + (x - 1) \times (15x^2 + 3)$$

$$= (5x^3 + 3x - 1) + (x - 1)(15x^2 + 3)$$

$$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3$$

$$= 20x^3 - 15x^2 + 6x - 4$$

(iii)

$$\text{Let } f(x) = x^{-3}(5 + 3x)$$

By differentiating both sides and using Leibnitz product rule, we get

$$f'(x) = x^{-3} \frac{d}{dx}(5 + 3x) + (5 + 3x) \frac{d}{dx}(x^{-3})$$

$$= x^{-3}(0 + 3) + (5 + 3x)(-3x^{-3-1})$$

By further calculation, we get

$$= x^{-3}(3) + (5 + 3x)(-3x^{-4})$$

$$= 3x^{-3} - 15x^{-4} - 9x^{-3}$$

$$= -6x^{-3} - 15x^{-4}$$

$$= -3x^{-3} \left(2 + \frac{5}{x} \right)$$

$$= \frac{-3x^{-3}}{x}(2x+5)$$

$$= \frac{-3}{x^4}(5+2x)$$

(iv)

$$\text{Let } f(x) = x^5(3 - 6x^{-9})$$

By differentiating both sides and using Leibnitz product rule, we get

$$f'(x) = x^5 \frac{d}{dx}(3 - 6x^{-9}) + (3 - 6x^{-9}) \frac{d}{dx}(x^5)$$

$$= x^5 \{0 - 6(-9)x^{-9-1}\} + (3 - 6x^{-9})(5x^4)$$

By further calculation, we get

$$= x^5(54x^{-10}) + 15x^4 - 30x^{-5}$$

$$= 54x^{-5} + 15x^4 - 30x^{-5}$$

$$= 24x^{-5} + 15x^4$$

$$= 15x^4 + \frac{24}{x^5}$$

(v)

$$\text{Let } f(x) = x^{-4}(3 - 4x^{-5})$$

By differentiating both sides and using Leibnitz product rule, we get

$$f'(x) = x^{-4} \frac{d}{dx}(3 - 4x^{-5}) + (3 - 4x^{-5}) \frac{d}{dx}(x^{-4})$$

$$= x^{-4} \{0 - 4(-5)x^{-5-1}\} + (3 - 4x^{-5})(-4)x^{-4-1}$$

By further calculation, we get

$$= x^{-4}(20x^{-6}) + (3 - 4x^{-5})(-4x^{-5})$$

$$= 20x^{-10} - 12x^{-5} + 16x^{-10}$$

$$= 36x^{-10} - 12x^{-5}$$

$$= -\frac{12}{x^5} + \frac{36}{x^{10}}$$

(vi)

$$\text{Let } f(x) = \frac{2}{x+1} - \frac{x^2}{3x-1}$$

By differentiating both sides we get,

$$f'(x) = \frac{d}{dx} \left(\frac{2}{x+1} - \frac{x^2}{3x-1} \right)$$

Using quotient rule we get,

$$f'(x) = \left[\frac{(x+1) \frac{d}{dx}(2) - 2 \frac{d}{dx}(x+1)}{(x+1)^2} \right] - \left[\frac{(3x-1) \frac{d}{dx}(x^2) - x^2 \frac{d}{dx}(3x-1)}{(3x-1)^2} \right]$$

$$= \left[\frac{(x+1)(0) - 2(1)}{(x+1)^2} \right] - \left[\frac{(3x-1)(2x) - (x^2) \times 3}{(3x-1)^2} \right]$$

$$= -\frac{2}{(x+1)^2} - \left[\frac{6x^2 - 2x - 3x^2}{(3x-1)^2} \right]$$

$$= -\frac{2}{(x+1)^2} - \frac{x(3x-2)}{(3x-1)^2}$$

Question 10

Find the derivative of $\cos x$ from the first principle.

Solution

Let $f(x) = \cos x$

Accordingly, $f(x + h) = \cos(x + h)$

By first principle, we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

So, we get

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{1}{h} [\cos(x + h) - \cos(x)] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[-2 \sin\left(\frac{x + h + x}{2}\right) \sin\left(\frac{x + h - x}{2}\right) \right] \end{aligned}$$

By further calculation, we get

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{1}{h} \left[-2 \sin\left(\frac{2x + h}{2}\right) \sin\left(\frac{h}{2}\right) \right] \\ &= \lim_{h \rightarrow 0} -\sin\left(\frac{2x + h}{2}\right) \times \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \\ &= -\sin\left(\frac{2x + 0}{2}\right) \times 1 \\ &= -\sin(2x / 2) \\ &= -\sin(x) \end{aligned}$$

Question 11

Find the derivative of the following functions.

(i) $\sin x \cos x$

(ii) $\sec x$

(iii) $5 \sec x + 4 \cos x$

(iv) cosec x

(v) 3 cot x + 5 cosec x

(vi) 5 sin x – 6 cos x + 7

(vii) 2 tan x – 7 sec x

Solution

(i) sin x cos x

Let $f(x) = \sin x \cos x$

Accordingly, from the first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(x+h) \cos(x+h) - \sin x \cos x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{2h} [2 \sin(x+h) \cos(x+h) - 2 \sin x \cos x] \\
 &= \lim_{h \rightarrow 0} \frac{1}{2h} [\sin 2(x+h) - \sin 2x] \\
 &= \lim_{h \rightarrow 0} \frac{1}{2h} \left[2 \cos \frac{2x+2h+2x}{2} \cdot \sin \frac{2x+2h-2x}{2} \right]
 \end{aligned}$$

By further calculation, we get

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\cos \frac{4x+2h}{2} \sin \frac{2h}{2} \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} [\cos(2x+h) \sin h] \\
 &= \lim_{h \rightarrow 0} \cos(2x+h) \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h} \\
 &= \cos(2x+0) \cdot 1 \\
 &= \cos 2x
 \end{aligned}$$

(ii) sec x

Let $f(x) = \sec x$

$$= 1 / \cos x$$

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx} \left(\frac{1}{\cos x} \right)$$

Using quotient rule, we get

$$\begin{aligned} f'(x) &= \frac{\cos x \frac{d}{dx}(1) - 1 \frac{d}{dx}(\cos x)}{\cos^2 x} \\ &= \frac{\cos x \times 0 - (-\sin x)}{\cos^2 x} \end{aligned}$$

We get

$$\begin{aligned} &= \frac{\sin x}{\cos^2 x} \\ &= \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \\ &= \tan x \sec x \end{aligned}$$

(iii) $5 \sec x + 4 \cos x$

Let $f(x) = 5 \sec x + 4 \cos x$

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx}(5 \sec x + 4 \cos x)$$

By further calculation, we get

$$= 5 \frac{d}{dx}(\sec x) + 4 \frac{d}{dx}(\cos x)$$

$$= 5 \sec x \tan x + 4 \times (-\sin x)$$

$$= 5 \sec x \tan x - 4 \sin x$$

(iv) cosec x

Let $f(x) = \operatorname{cosec} x$

Accordingly $f(x+h) = \operatorname{cosec}(x+h)$

By first principle, we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\operatorname{cosec}(x+h) - \operatorname{cosec} x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{\sin(x+h)} - \frac{1}{\sin x} \right)$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin x - \sin(x+h)}{\sin x \sin(x+h)} \right] \\
&= \frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{2 \cos\left(\frac{x+x+h}{2}\right) \sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)} \right] \\
&= \frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{-h}{2}\right)}{\sin(x+h)} \right]
\end{aligned}$$

By further calculation, we get

$$\begin{aligned}
&= \frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-\sin\left(\frac{h}{2}\right) \cos\left(\frac{2x+h}{2}\right)}{\left(\frac{h}{2}\right) \sin(x+h)} \right] \\
&= -\frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \rightarrow 0} \frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)} \\
&= -\frac{1}{\sin x} \times 1 \times \frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)} \\
&= -\frac{1}{\sin x} \times \frac{\cos x}{\sin x} \\
&= -\operatorname{cosec} x \cot x
\end{aligned}$$

(v) $3 \cot x + 5 \operatorname{cosec} x$

Let $f(x) = 3 \cot x + 5 \operatorname{cosec} x$

$$f'(x) = 3 (\cot x)' + 5 (\operatorname{cosec} x)'$$

Let $f_1(x) = \cot x$,

Accordingly $f_1(x+h) = \cot(x+h)$

By using first principle, we get

$$f_1'(x) = \lim_{h \rightarrow 0} \frac{f_1(x+h) - f_1(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cot(x+h) - \cot x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x} \right)$$

By further calculation, we get

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{\sin x \cos(x+h) - \cos x \sin(x+h)}{\sin x \sin(x+h)} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{\sin(x-x-h)}{\sin x \sin(x+h)} \right)$$

$$= \frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin(-h)}{\sin(x+h)} \right]$$

$$= -\frac{1}{\sin x} \left(\lim_{h \rightarrow 0} \frac{\sin h}{h} \right) \left(\lim_{h \rightarrow 0} \frac{1}{\sin(x+h)} \right)$$

$$= -\frac{1}{\sin x} \times 1 \times \frac{1}{\sin(x+0)}$$

$$= -\frac{1}{\sin^2 x}$$

$$= -\operatorname{cosec}^2 x$$

Let $f_2(x) = \operatorname{cosec} x$,

Accordingly $f_2(x+h) = \operatorname{cosec}(x+h)$

By using first principle, we get

$$f_2'(x) = \lim_{h \rightarrow 0} \frac{f_2(x+h) - f_2(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\operatorname{cosec}(x+h) - \operatorname{cosec} x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{\sin(x+h)} - \frac{1}{\sin x} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin x - \sin(x+h)}{\sin x \sin(x+h)} \right]$$

By further calculation, we get

$$= \frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{2 \cos\left(\frac{x+x+h}{2}\right) \sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)} \right]$$

$$= \frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{-h}{2}\right)}{\sin(x+h)} \right]$$

$$= \frac{1}{\sin x} \lim_{h \rightarrow 0} \left[\frac{-\sin\left(\frac{h}{2}\right) \cos\left(\frac{2x+h}{2}\right)}{\left(\frac{h}{2}\right) \sin(x+h)} \right]$$

$$= -\frac{1}{\sin x} \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \rightarrow 0} \frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)}$$

$$= -\frac{1}{\sin x} \times 1 \times \frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)}$$

$$= -\frac{1}{\sin x} \times \frac{\cos x}{\sin x}$$

$$= -\operatorname{cosec} x \cot x$$

Now, substitute the value of $(\cot x)'$ and $(\operatorname{cosec} x)'$ in $f'(x)$, we get

$$f'(x) = 3 (\cot x)' + 5 (\operatorname{cosec} x)'$$

$$f'(x) = 3 \times (-\operatorname{cosec}^2 x) + 5 \times (-\operatorname{cosec} x \cot x)$$

$$f'(x) = -3\operatorname{cosec}^2 x - 5\operatorname{cosec} x \cot x$$

$$(vi) 5 \sin x - 6 \cos x + 7$$

$$\text{Let } f(x) = 5 \sin x - 6 \cos x + 7$$

Accordingly, from the first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} [5 \sin(x+h) - 6 \cos(x+h) + 7 - 5 \sin x + 6 \cos x - 7] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} [5 \{\sin(x+h) - \sin x\} - 6 \{\cos(x+h) - \cos x\}] \\
 &= 5 \lim_{h \rightarrow 0} \frac{1}{h} [\sin(x+h) - \sin x] - 6 \lim_{h \rightarrow 0} \frac{1}{h} [\cos(x+h) - \cos x]
 \end{aligned}$$

By further calculation, we get

$$\begin{aligned}
 &= 5 \lim_{h \rightarrow 0} \frac{1}{h} \left[2 \cos\left(\frac{x+h+x}{2}\right) \sin\left(\frac{x+h-x}{2}\right) \right] - 6 \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\
 &= 5 \lim_{h \rightarrow 0} \frac{1}{h} \left[2 \cos\left(\frac{2x+h}{2}\right) \sin \frac{h}{2} \right] - 6 \lim_{h \rightarrow 0} \left[\frac{-\cos x (1 - \cos h) - \sin x \sin h}{h} \right]
 \end{aligned}$$

Now, we get

$$\begin{aligned}
 &= 5 \lim_{h \rightarrow 0} \left(\cos\left(\frac{2x+h}{2}\right) \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) - 6 \lim_{h \rightarrow 0} \left[\frac{-\cos x (1 - \cos h)}{h} - \frac{\sin x \sin h}{h} \right] \\
 &= 5 \left[\lim_{h \rightarrow 0} \cos\left(\frac{2x+h}{2}\right) \right] \left[\lim_{\frac{h}{2} \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right] - 6 \left[(-\cos x) \left(\lim_{h \rightarrow 0} \frac{1 - \cos h}{h} \right) - \sin x \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right) \right] \\
 &= 5 \cos x \cdot 1 - 6 [(-\cos x) \cdot (0) - \sin x \cdot 1] \\
 &= 5 \cos x + 6 \sin x
 \end{aligned}$$

(vii) $2 \tan x - 7 \sec x$

Let $f(x) = 2 \tan x - 7 \sec x$

Accordingly, from the first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} [2 \tan(x+h) - 7 \sec(x+h) - 2 \tan x + 7 \sec x] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} [2 \{ \tan(x+h) - \tan x \} - 7 \{ \sec(x+h) - \sec x \}] \\
 &= 2 \lim_{h \rightarrow 0} \frac{1}{h} [\tan(x+h) - \tan x] - 7 \lim_{h \rightarrow 0} \frac{1}{h} [\sec(x+h) - \sec x]
 \end{aligned}$$

By further calculation, we get

$$\begin{aligned}
 &= 2 \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x} \right] - 7 \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{1}{\cos(x+h)} - \frac{1}{\cos x} \right] \\
 &= 2 \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin(x+h) \cos x - \sin x \cos(x+h)}{\cos x \cos(x+h)} \right] - 7 \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\cos x - \cos(x+h)}{\cos x \cos(x+h)} \right] \\
 &= 2 \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin(x+h-x)}{\cos x \cos(x+h)} \right] - 7 \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-2 \sin\left(\frac{x+x+h}{2}\right) \sin\left(\frac{x-x-h}{2}\right)}{\cos x \cos(x+h)} \right]
 \end{aligned}$$

Now, we get

$$\begin{aligned}
 &= 2 \lim_{h \rightarrow 0} \left[\left(\frac{\sin h}{h} \right) \frac{1}{\cos x \cos(x+h)} \right] - 7 \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-2 \sin\left(\frac{2x+h}{2}\right) \sin\left(-\frac{h}{2}\right)}{\cos x \cos(x+h)} \right] \\
 &= 2 \left(\lim_{h \rightarrow 0} \frac{\sin h}{h} \right) \left(\lim_{h \rightarrow 0} \frac{1}{\cos x \cos(x+h)} \right) - 7 \left(\lim_{\frac{h}{2} \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) \left(\lim_{h \rightarrow 0} \frac{\sin\left(\frac{2x+h}{2}\right)}{\cos x \cos(x+h)} \right) \\
 &= 2 \cdot 1 \cdot \frac{1}{\cos x \cos x} - 7 \cdot 1 \left(\frac{\sin x}{\cos x \cos x} \right) \\
 &= 2 \sec^2 x - 7 \sec x \tan x
 \end{aligned}$$