

Chapter 13: Limits and Derivatives Exercise 13.1

NCERT · Class 11 · Maths

Exercise 13.1

Question 1

Evaluate the Given limit. $\lim_{x \rightarrow 3} x + 3$

Solution

Ans. Given,

$$\lim_{x \rightarrow 3} x + 3$$

$$= 3 + 3$$

$$= 6$$

Question 2

Evaluate the Given limit. $\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$

Solution

Ans. Given,

$$\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$$

$$=$$

$$\left(\pi - \frac{22}{7} \right)$$

Question 3

Evaluate the Given limit. $\lim_{r \rightarrow 1} \pi r^2$

Solution

Ans. Given,

$$\lim_{r \rightarrow 1} \pi r^2$$

=

$$\pi(1^2)$$

=

$$\pi$$

Question 4

Evaluate the Given limit.

$$\lim_{x \rightarrow 1} \frac{4x + 3}{x - 2}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 1} \frac{4x + 3}{x - 2}$$

$$= \frac{4(4)+3}{4-2}$$

$$= \frac{16+3}{2}$$

$$= \frac{19}{2}$$

Question 5

Evaluate the Given limit.

$$\lim_{x \rightarrow 1} \frac{x^{10} + x^5 + 1}{x - 1}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 1} \frac{x^{10} + x^5 + 1}{x - 1}$$

$$= \frac{(-1)^{10} + (-1)^5 + 1}{-1 - 1}$$

$$= \frac{1 - 1 + 1}{-2}$$

$$= -\frac{1}{2}$$

Question 6

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{(x + 1)^5 - 1}{x}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{(x + 1)^5 - 1}{x}$$

Put $x + 1 = y$ So,

$y \rightarrow 1$ as $x \rightarrow 0$

Than,

$$\lim_{x \rightarrow 0} \frac{(x + 1)^5 - 1}{x}$$

=

$$\lim_{x \rightarrow 1} \frac{(y)^5 - 1}{y - 1}$$

Using

$$\left[\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= 5 \cdot 1^{5-1}$$

$$= 5$$

Question 7

Evaluate the Given limit.

$$\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

SolutionAns. At $x = 2$

The value of the given rational function takes the form

$$\frac{0}{0}$$

=

$$\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

=

$$\lim_{x \rightarrow 2} \frac{(x - 2)(3x + 5)}{(x - 2)(x + 2)}$$

=

$$\lim_{x \rightarrow 2} \frac{3x + 5}{x + 2}$$

=

$$\frac{3(2) + 5}{2 + 2}$$

$$= \frac{11}{4}$$

Question 8

Evaluate the Given limit.

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

Solution

Ans. At $x = 2$

The value of the given rational function takes the form

$$\frac{0}{0}$$

=

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

=

$$\lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)(x^2 + 9)}{(x - 3)(2x + 1)}$$

=

$$\lim_{x \rightarrow 3} \frac{(x+3)(x^2+9)}{(2x+1)}$$

=

$$\frac{(3+3)(3^2+9)}{(2(3)+1)}$$

=

$$\frac{6 \times 18}{7}$$

=

$$\frac{108}{7}$$

Question 9

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{ax+b}{cx+1}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{ax+b}{cx+1}$$

=

$$\frac{a(0)+b}{c(0)+1}$$

=

$$b$$

Question 10**Evaluate the Given limit.**

$$\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

SolutionAns. At $z = 1$

The value of the given rational function takes the form

$$\frac{0}{0}$$

Put $z^{\frac{1}{6}} = x$ So, $z \rightarrow 1$ as $x \rightarrow 1$

Than,

$$\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

=

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

Using

$$\left[\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= 2 \cdot 1^{2-1}$$

$$= 2$$

Question 11

Evaluate the Given limit.

$$\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}$$

,

$$a + b + c \neq 0$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}$$

=

$$\frac{a(1)^2 + b(1) + c}{c(1)^2 + b(1) + a}$$

=

$$\frac{a + b + c}{c + b + a}$$

=1

$$(a + b + c \neq 0)$$

Question 12

Evaluate the Given limit.

$$\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$$

At $x = -2$

The value of the given rational function takes the form

$$\frac{0}{0}$$

$$\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$$

=

$$\lim_{x \rightarrow -2} \frac{\left(\frac{2+x}{2x}\right)}{x + 2}$$

=

$$\lim_{x \rightarrow -2} \frac{1}{2x}$$

=

$$\frac{1}{2(-2)}$$

=

$$-\frac{1}{4}$$

Question 13

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$$

At $x = 0$

The value of the given rational function takes the form

$$\frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \lim_{x \rightarrow 0} \frac{\sin ax}{bx} \times \frac{ax}{ax}$$

=

$$\lim_{x \rightarrow 0} \left(\frac{\sin ax}{bx} \right) \times \frac{a}{b}$$

=

$$\frac{a}{b} \lim_{ax \rightarrow 0} \left(\frac{\sin ax}{bx} \right)$$

$$x \rightarrow 0 \Rightarrow ax \rightarrow 0$$

=

$$\frac{a}{b} \times 1$$

$$\left[\lim_{x \rightarrow 0} \left(\frac{\sin y}{y} \right) \right]$$

=

$$\frac{a}{b}$$

Question 14

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$$

At $x = 0$

The value of the given rational function takes the form

$$\frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax} \right) \times ax}{\left(\frac{\sin bx}{bx} \right) \times bx}$$

=

$$\frac{a}{b} \times \frac{\lim_{ax \rightarrow 0} \left(\frac{\sin ax}{ax} \right)}{\lim_{bx \rightarrow 0} \left(\frac{\sin bx}{bx} \right)}$$

$$\left[\begin{array}{l} x \rightarrow 0 \Rightarrow ax \rightarrow 0 \\ x \rightarrow 0 \Rightarrow bx \rightarrow 0 \end{array} \right]$$

=

$$\frac{a}{b} \times \frac{1}{1}$$

$$\left[\lim_{x \rightarrow 0} \left(\frac{\sin y}{y} \right) = 1 \right]$$

=

$$\frac{a}{b}$$

Question 15

Evaluate the Given limit.

$$\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

$$[x \rightarrow \pi \Rightarrow (\pi - x) \rightarrow 0]$$

$$\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \frac{1}{\pi} \lim_{(\pi - x) \rightarrow 0} \frac{\sin(\pi - x)}{(\pi - x)}$$

$$= \frac{1}{\pi} \times 1 \quad \left[\lim_{x \rightarrow 0} \left(\frac{\sin y}{y} \right) = 1 \right]$$

$$= \frac{1}{\pi}$$

Question 16

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x}$$

=

$$\frac{\cos 0}{\pi - 0}$$

=

$$\frac{1}{\pi}$$

Question 17

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$$

At $x = 0$

The value of the given rational function takes the form

$$\frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \rightarrow 0} \frac{1 - 2\sin^2 x - 1}{1 - 2\sin^2 \frac{x}{2} - 1}$$

=

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \frac{\left(\frac{\sin^2 x}{x^2}\right) \times x^2}{\left(\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right) \times \frac{x^2}{4}}$$

=

$$4 \frac{\lim_{x \rightarrow 0} \left(\frac{\sin^2 x}{x^2} \right)}{\lim_{x \rightarrow 0} \left(\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2} \right)^2} \right)}$$

=

$$4 \frac{\left(\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \right)^2}{\left(\lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2} \right)^2} \right)^2}$$

$$\left[x \rightarrow 0 \Rightarrow \frac{x}{2} \rightarrow 0 \right]$$

=

$$4 \frac{1^2}{1^2}$$

$$\left[\lim_{x \rightarrow 0} \left(\frac{\sin y}{y} \right) = 1 \right]$$

=

$$4$$

Question 18

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$$

At $x = 0$

The value of the given rational function takes the form

$$\frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x} = \frac{1}{b} \lim_{x \rightarrow 0} \frac{x(a + \cos x)}{\sin x}$$

$$\frac{1}{b} \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \right) \times \lim_{x \rightarrow 0} (a + \cos x)$$

=

$$\frac{1}{b} \lim_{x \rightarrow 0} \left(\frac{1}{\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)} \right) \times \lim_{x \rightarrow 0} (a + \cos x)$$

=

$$\frac{1}{b} \times (a + \cos 0)$$

$$\left[\lim_{y \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right]$$

=

$$\frac{a + 1}{b}$$

Question 19

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} x \sec x$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} x \sec x$$

=

$$\lim_{x \rightarrow 0} \frac{x}{\cos x}$$

=

$$\frac{0}{\cos 0}$$

=

$$\frac{0}{1}$$

=

$$0$$

Question 20

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}$$

$$a, b, a + b \neq 0$$

Solution

Ans. At $x = 0$

The value of the given rational function takes the form

$$\frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}$$

=

$$\lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax}\right) ax + bx}{ax + bx \left(\frac{\sin bx}{bx}\right)}$$

=

$$\frac{\left(\lim_{x \rightarrow 0} \frac{\sin ax}{ax}\right) \times \lim_{x \rightarrow 0} (ax) + \lim_{x \rightarrow 0} (bx)}{\lim_{x \rightarrow 0} ax + \lim_{x \rightarrow 0} bx \left(\lim_{x \rightarrow 0} \frac{\sin bx}{bx}\right)}$$

$$[x \rightarrow 0 \Rightarrow ax \rightarrow 0]$$

and

$$[bx \rightarrow 0]$$

=

$$\frac{\lim_{x \rightarrow 0} (ax) + \lim_{x \rightarrow 0} (bx)}{\lim_{x \rightarrow 0} ax + \lim_{x \rightarrow 0} bx}$$

$$\left[\lim_{y \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right]$$

=

$$\frac{\lim_{x \rightarrow 0} (ax + bx)}{\lim_{x \rightarrow 0} (ax + bx)}$$

=

$$\lim_{x \rightarrow 0} (1)$$

=

$$1$$

Question 21

Evaluate the Given limit.

$$\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$$

Solution

Ans. At $x = 0$

The value of the given rational function takes the form

$$\infty \rightarrow \infty$$

$$\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$$

=

$$\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

=

$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\sin x} \right)$$

=

$$\lim_{x \rightarrow 0} \frac{\left(\frac{1 - \cos x}{x} \right)}{\left(\frac{\sin x}{x} \right)}$$

=

$$\frac{\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x} \right)}{\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)}$$

$$\left[\lim_{y \rightarrow 0} \frac{1 - \cos x}{x} = 0 \right] \text{ and } \left[\lim_{y \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right]$$

=

$$\frac{0}{1}$$

=

$$0$$

Question 22

Evaluate the Given limit.

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

Solution

Ans. Given,

$$\lim_{x \rightarrow 0} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

At $x = \frac{\pi}{2}$

The value of the given rational function takes the form

$$\frac{0}{0}$$

Put $x - \frac{\pi}{2} = y$

So,

$$\left[x \rightarrow \frac{\pi}{2}, y \rightarrow 0 \right]$$

$$\lim_{x \rightarrow 0} \frac{\tan 2x}{x - \frac{\pi}{2}} = \lim_{y \rightarrow 0} \frac{\tan 2 \left(y + \frac{\pi}{2} \right)}{y}$$

=

$$\lim_{y \rightarrow 0} \frac{\tan(\pi + 2y)}{y}$$

=

$$\lim_{y \rightarrow 0} \frac{\tan 2y}{y}$$

$$[\tan(\pi + 2y) = \tan 2y]$$

=

$$\lim_{y \rightarrow 0} \frac{\sin 2y}{y \cos 2y}$$

=

$$\lim_{y \rightarrow 0} \left(\frac{\sin 2y}{2y} \times \frac{2}{\cos 2y} \right)$$

=

$$\lim_{y \rightarrow 0} \left(\frac{\sin 2y}{2y} \right) \times \lim_{y \rightarrow 0} \left(\frac{2}{\cos 2y} \right)$$

$$[y \rightarrow 0 \Rightarrow 2y \rightarrow 0]$$

=

$$1 \times \frac{2}{\cos 0}$$

$$\left[\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right]$$

=

$$1 \times \frac{2}{1}$$

=

$$2$$

Question 23

Find $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$, where

$$f(x) = \begin{cases} 2x + 3 \\ 3(x + 1) \end{cases}$$

$$x \leq 0$$

$$x > 0$$

Solution

Ans. Given.

$$f(x) = \begin{cases} 2x + 3 \\ 3(x + 1) \end{cases}$$

$$x \leq 0$$

$$x > 0$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} [2x + 3]$$

=

$$2(0) + 3 = 3$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} [3x + 1]$$

=

$$3(0 + 1) = 3$$

Therefore,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} f(x) = 3$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} (x + 1) = 3(1 + 1) = 6$$

Therefore,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} f(x) = 6$$

Question 24

Find $\lim_{x \rightarrow 1} f(x)$, where

$$f(x) = \begin{cases} x^2 - 1 \\ -x - 1 \end{cases}$$

$$x \leq 1$$

$$x > 1$$

Solution

Ans. Given,

$$f(x) = \begin{cases} x^2 - 1 \\ -x - 1 \end{cases}$$

$$\begin{cases} x \leq 1 \\ x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} (x^2 - 1) = 1^2 - 1 = 1 - 1 = 0$$

So, it is observed that

$$\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

Hence, $\lim_{x \rightarrow 1} f(x)$ does not exist.

Question 25

Find $\lim_{x \rightarrow 0} f(x)$, where

$$f(x) = \begin{cases} \frac{|x|}{x} \\ 0 \end{cases}$$

$$\begin{cases} x \neq 0 \\ x = 0 \end{cases}$$

Solution

Ans. Given

$$f(x) = \begin{cases} \frac{|x|}{x} \\ 0 \end{cases}$$

$$\begin{cases} x \neq 0 \\ x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \left[\frac{|x|}{x} \right]$$

=

$$\lim_{x \rightarrow 0} \left(\frac{-x}{x} \right)$$

when x is negative, $|x| = -x$

=

$$\lim_{x \rightarrow 0} (-1)$$

=

$$-1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left[\frac{|x|}{x} \right]$$

$$= \lim_{x \rightarrow 0} \left(\frac{x}{x} \right)$$

when x is positive, $|x| = x$

$$= \lim_{x \rightarrow 0} (1)$$

=

$$1$$

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Hence, $\lim_{x \rightarrow 0} f(x)$ does not exist.

Question 26

Find $\lim_{x \rightarrow 0} f(x)$, where

$$f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$x \neq 0$$

$$x = 0$$

Solution

Ans. Given

$$f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$x \neq 0$$

$$x = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \left[\frac{x}{|x|} \right]$$

=

$$\lim_{x \rightarrow 0} \left(\frac{-x}{x} \right)$$

when x is negative, $|x| = -x$

=

$$\lim_{x \rightarrow 0} (-1)$$

=

$$-1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left[\frac{x}{|x|} \right]$$

$$= \lim_{x \rightarrow 0} \left(\frac{x}{x} \right)$$

when x is positive, $|x| = x$

=

$$\lim_{x \rightarrow 0} (1)$$

=

$$1$$

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Hence, $\lim_{x \rightarrow 0} f(x)$ does not exist.

Question 27

Find $\lim_{x \rightarrow 5} f(x)$, where $f(x) = |x| - 5$

Solution

Ans. Given,

$$f(x) = |x| - 5$$

$$\begin{aligned}
 \lim_{x \rightarrow 5^-} f(x) &= \lim_{x \rightarrow 5^-} (|x| - 5) \\
 &= \lim_{x \rightarrow 5} (x - 5) \quad \text{when } x \text{ is positive, } |x| = x \\
 &= 5 - 5 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 5^+} f(x) &= \lim_{x \rightarrow 5^+} (|x| - 5) \\
 &= \lim_{x \rightarrow 5} (x - 5) \\
 &\text{when } x \text{ is positive, } |x| = x \\
 &= 5 - 5 \\
 &= 0
 \end{aligned}$$

$$\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x)$$

$$\text{Hence, } \lim_{x \rightarrow 5} f(x) = 0$$

Question 28

Suppose $f(x) = \begin{cases} a + bx & x < 0 \\ 4 & x = 1 \\ b - ax & x > 1 \end{cases}$ and if $\lim_{x \rightarrow 1} f(x) = f(1)$ what are possible values of a and b?

Solution

Ans. The given function is

$$f(x) = \begin{cases} a + bx & x < 0 \\ 4 & x = 1 \\ b - ax & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} f(ax + bx) = a + b$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} f(b - ax) = b - a$$

$$f(1) = 4$$

Given

$$\lim_{x \rightarrow 1} f(x) = f(1)$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x) = f(1)$$

$$a + b = 4 \text{ and } b - a = 4$$

On solving, we get

$$a = 0 \text{ and } b = 4$$

Question 29

Let $a_1, a_2, \dots, a_n$ be fixed real number and define a function

$$f(x) = (x - a_1)(x - a_2) \dots (x - a_n)$$

What is $\lim_{x \rightarrow a_1} f(x)$? For some $a \neq a_1, a_2, \dots, a_n$. Compute $\lim_{x \rightarrow a} f(x)$.

Solution

Ans. Given,

$$f(x) = (x - a_1)(x - a_2) \dots (x - a_n)$$

$$\lim_{x \rightarrow a_1} f(x) = \lim_{x \rightarrow a_1} [(x - a_1)(x - a_2) \dots (x - a_n)]$$

=

$$(a_1 - a_1)(a_1 - a_2) \dots (a_1 - a_n) = 0$$

Therefore,

$$\lim_{x \rightarrow a_1} f(x) = 0$$

$$\text{Now, } \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} [(x - a_1)(x - a_2) \dots (x - a_n)]$$

=

$$(a_1 - a_1)(a_1 - a_2) \dots (a_1 - a_n)$$

Therefore,

$$\lim_{x \rightarrow a} f(x) = (a_1 - a_1)(a_1 - a_2) \dots (a_1 - a_n)$$

Question 30

If

$$f(x) = \begin{cases} |x| + 1 \\ 0 \\ |x| - 1 \end{cases}$$

$$x < 0$$

$$x = 0$$

$$x > 1$$

For what value (s) of does $\lim_{x \rightarrow a} f(x)$ exists?

Solution

Ans. Given,

$$f(x) = \begin{cases} |x| + 1 \\ 0 \\ |x| - 1 \end{cases}$$

$$x < 0$$

$$x = 0$$

$$x > 1$$

When $a = 0$

=

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (|x| + 1)$$

=

$$\lim_{x \rightarrow 0^-} (-x + 1)$$

when x is negative, $|x| = -x$

$$= 0 + 1$$

$$= 1$$

=

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (|x| + 1)$$

=

$$\lim_{x \rightarrow 0^-} (x - 1)$$

when x is positive, $|x| = x$

$$= 0 - 1$$

$$= -1$$

Here, $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$

$\lim_{x \rightarrow 0} f(x)$ does not exist.

When

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^+} (|x| + 1)$$

$$a < 0$$

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^-} (|x| + 1)$$

=

$$\lim_{x \rightarrow a^-} (-x + 1)$$

$$[x < a < 0 \Rightarrow |x| = -x]$$

=

$$-a + 1$$

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = -a + 1$$

Thus, limit exists at $x = a$, where

$$a < 0$$

When

$$a > 0$$

$$\begin{aligned}
 \lim_{x \rightarrow a^-} f(x) &= \lim_{x \rightarrow a^-} (|x| + 1) \\
 &= \lim_{x \rightarrow a} (-x - 1) \quad [0 < x < a \Rightarrow |x| = x] \\
 &= -a - 1
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow a^+} f(x) &= \lim_{x \rightarrow a^+} (|x| + 1) \\
 &= \lim_{x \rightarrow a} (-x - 1) \quad [0 < x < a \Rightarrow |x| = x] \\
 &= a - 1
 \end{aligned}$$

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = a - 1$$

Thus, limit exists at $x = a$, where

$$a > 0$$

Thus $\lim_{x \rightarrow a} f(x)$ exists for all $a \neq 0$ $\lim_{x \rightarrow 1} \frac{f(x)-2}{x^2-1} = \pi$.

Question 31

If function $f(x)$ satisfies,

$$\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi$$

, evaluate $\lim_{x \rightarrow 1} f(x)$

Solution

Ans. Given,

$$\begin{aligned}
 &\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi \\
 &= \frac{\lim_{x \rightarrow 1} (f(x) - 2)}{\lim_{x \rightarrow 1} (x^2 - 1)} = \pi \\
 &= \lim_{x \rightarrow 1} (f(x) - 2) = \pi \lim_{x \rightarrow 1} (x^2 - 1) \\
 &= \lim_{x \rightarrow 1} (f(x) - 2) = \pi(1^2 - 1) \\
 &= \lim_{x \rightarrow 1} (f(x) - 2) = 0 \\
 &= \lim_{x \rightarrow 1} f(x) - \lim_{x \rightarrow 1} 2 = 0
 \end{aligned}$$

$$= \lim_{x \rightarrow 1} f(x) - 2 = 0$$

$$\lim_{x \rightarrow 1} f(x) = 2$$

Question 32

$$\text{If } f(x) = \begin{cases} mx^2 + n & x < 0 \\ nx + m & 0 \leq x \leq 1 \\ nx^3 + m & x > 1 \end{cases}$$

For what integers m and n does $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$ exists?

Solution

Ans. Given,

$$f(x) = \begin{cases} mx^2 + n & x < 0 \\ nx + m & 0 \leq x \leq 1 \\ nx^3 + m & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (mx^2 + n)$$

$$= m(0)^2 + n$$

$$= n$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} (nx + m)$$

$$= n(0) + m$$

$$= m$$

Thus, $\lim_{x \rightarrow 0^+} f(x)$ exists if $m=n$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} (nx + m)$$

$$= n(1) + m$$

$$= m + n$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} (nx^3 + m)$$

$$= n(1)^3 + m$$

$$=m + n$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x)$$

Thus, $\lim_{x \rightarrow 1} f(x)$ exists for any internal value of m and n
