

## Chapter 12: Introduction to Three Dimensional Geometry

### Exercise 12.3

NCERT · Class 11 · Maths

#### Questions

##### Question 1

Find the coordinates of the point which divides the line segment joining the points  $(-2, 3, 5)$  and  $(1, -4, 6)$  in the ratio (i) 2: 3 internally, (ii) 2: 3 externally.

##### Solution

Let the line segment joining the points P  $(-2, 3, 5)$  and Q  $(1, -4, 6)$  be PQ.

(i) 2: 3 internally

By using the section formula,

We know that the coordinates of the point R, which divide the line segment joining two points, P  $(x_1, y_1, z_1)$  and Q  $(x_2, y_2, z_2)$ , internally in the ratio  $m: n$ , are given by

$$\left( \frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n} \right)$$

Upon comparing, we have

$$x_1 = -2, y_1 = 3, z_1 = 5;$$

$$x_2 = 1, y_2 = -4, z_2 = 6 \text{ and}$$

$$m = 2, n = 3$$

So, the coordinates of the point which divide the line segment joining the points P  $(-2, 3, 5)$  and Q  $(1, -4, 6)$  in the ratio 2 : 3 internally are given by

$$\begin{aligned}
 & \left( \frac{2 \times 1 + 3 \times (-2)}{2 + 3}, \frac{2 \times (-4) + 3 \times 3}{2 + 3}, \frac{2 \times 6 + 3 \times 5}{2 + 3} \right) \\
 &= \left( \frac{2 - 6}{5}, \frac{-8 + 9}{5}, \frac{12 + 15}{5} \right) \\
 &= \left( \frac{-4}{5}, \frac{1}{5}, \frac{27}{5} \right)
 \end{aligned}$$

Hence, the coordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) is  $(-4/5, 1/5, 27/5)$ .

**(ii) 2: 3 externally**

By using the section formula,

We know that the coordinates of the point R, which divide the line segment joining two points, P ( $x_1, y_1, z_1$ ) and Q ( $x_2, y_2, z_2$ ), externally in the ratio m: n, are given by

$$\left( \frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}, \frac{mz_2 - nz_1}{m - n} \right)$$

Upon comparing, we have

$$x_1 = -2, y_1 = 3, z_1 = 5;$$

$$x_2 = 1, y_2 = -4, z_2 = 6 \text{ and}$$

$$m = 2, n = 3$$

So, the coordinates of the point which divide the line segment joining the points P (-2, 3, 5) and Q (1, -4, 6) in the ratio 2: 3 externally are given by

$$\begin{aligned}
 & \left( \frac{2 \times 1 - 3 \times (-2)}{2 - 3}, \frac{2 \times (-4) - 3 \times 3}{2 - 3}, \frac{2 \times 6 - 3 \times 5}{2 - 3} \right) \\
 &= \left( \frac{2 - (-6)}{-1}, \frac{-8 - 9}{-1}, \frac{12 - 15}{-1} \right) \\
 &= \left( \frac{8}{-1}, \frac{-17}{-1}, \frac{-3}{-1} \right) \\
 &= (-8, 17, 3)
 \end{aligned}$$

$\therefore$  The coordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) are (-8, 17, 3).

## Question 2

Given that P (3, 2, - 4), Q (5, 4, - 6) and R (9, 8, -10) are collinear. Find the ratio in which Q divides PR.

### Solution

Let us consider Q divides PR in the ratio k: 1.

By using the section formula,

We know that the coordinates of the point R, which divides the line segment joining two points, P ( $x_1$ ,  $y_1$ ,  $z_1$ ) and Q ( $x_2$ ,  $y_2$ ,  $z_2$ ), internally in the ratio m : n, are given by

$$\left( \frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n} \right)$$

Upon comparing, we have

$$x_1 = 3, y_1 = 2, z_1 = -4;$$

$$x_2 = 9, y_2 = 8, z_2 = -10 \text{ and}$$

$$m = k, n = 1$$

So, we have

$$\left( \frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1} \right) = (5, 4, -6)$$

$$\frac{9k + 3}{k + 1} = 5, \frac{8k + 2}{k + 1} = 4, \frac{-10k - 4}{k + 1} = -6$$

$$9k + 3 = 5(k + 1)$$

$$9k + 3 = 5k + 5$$

$$9k - 5k = 5 - 3$$

$$4k = 2$$

$$k = \frac{2}{4}$$

$$= \frac{1}{2}$$

Hence, the ratio in which Q divides PR is 1: 2.

### Question 3

Find the ratio in which the YZ-plane divides the line segment formed by joining the points  $(-2, 4, 7)$  and  $(3, -5, 8)$ .

### Solution

Let the line segment formed by joining the points P  $(-2, 4, 7)$  and Q  $(3, -5, 8)$  be PQ.

We know that any point on the YZ-plane is of the form  $(0, y, z)$ .

So now, let R  $(0, y, z)$  divide the line segment PQ in the ratio  $k: 1$ .

Then,

Upon comparing, we have

$$x_1 = -2, y_1 = 4, z_1 = 7$$

$$x_2 = 3, y_2 = -5, z_2 = 8 \text{ and}$$

$$m = k, n = 1$$

By using the section formula,

We know that the coordinates of the point R, which divide the line segment joining two points P  $(x_1, y_1, z_1)$  and Q  $(x_2, y_2, z_2)$  internally in the ratio  $m: n$  are given by

$$\left( \frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n} \right)$$

So we have,

$$\left( \frac{3k - 2}{k + 1}, \frac{-5k + 4}{k + 1}, \frac{8k + 7}{k + 1} \right) = (0, y, z)$$

$$\frac{3k - 2}{k + 1} = 0$$

$$3k - 2 = 0$$

$$3k = 2$$

$$k = 2/3$$

Hence, the ratio in which the YZ-plane divides the line segment formed by joining the points  $(-2, 4, 7)$  and  $(3, -5, 8)$  is 2:3.

#### Question 4

Using the section formula, show that the points A  $(2, -3, 4)$ , B  $(-1, 2, 1)$  and C  $(0, 1/3, 2)$  are collinear.

#### Solution

Let the point P divides AB in the ratio  $k:1$ .

Upon comparing, we have

$$x_1 = 2, y_1 = -3, z_1 = 4;$$

$$x_2 = -1, y_2 = 2, z_2 = 1 \text{ and}$$

$$m = k, n = 1$$

By using the section formula,

We know that the coordinates of the point R, which divide the line segment joining two points P  $(x_1, y_1, z_1)$  and Q  $(x_2, y_2, z_2)$  internally in the ratio  $m:n$  are given by

$$\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

So we have,

$$\text{The coordinates of P} = \left( \frac{-k+2}{k+1}, \frac{2k-3}{k+1}, \frac{k+4}{k+1} \right)$$

Now, we check if, for some value of  $k$ , the point coincides with point C.

$$\text{Put } (-k+2)/(k+1) = 0$$

$$-k + 2 = 0$$

$$k = 2$$

$$\text{When } k = 2, \text{ then } (2k-3)/(k+1) = (2(2)-3)/(2+1)$$

$$= (4-3)/3$$

$$= 1/3$$

$$\text{And, } (k+4)/(k+1) = (2+4)/(2+1)$$

$$= 6/3$$

$$= 2$$

∴ C (0, 1/3, 2) is a point which divides AB in the ratio 2: 1 and is the same as P.

Hence, A, B, C are collinear.

---

### Question 5

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

### Solution

Let A ( $x_1, y_1, z_1$ ) and B ( $x_2, y_2, z_2$ ) trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

A divides the line segment PQ in the ratio 1: 2.

Upon comparing, we have

$$x_1 = 4, y_1 = 2, z_1 = -6$$

$$x_2 = 10, y_2 = -16, z_2 = 6 \text{ and}$$

$$m = 1, n = 2$$

By using the section formula,

We know that the coordinates of the point R, which divide the line segment joining two points, P ( $x_1, y_1, z_1$ ) and Q ( $x_2, y_2, z_2$ ), internally in the ratio  $m : n$ , are given by

$$\left( \frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n} \right)$$

So we have,

$$\begin{aligned}\text{The coordinates of A} &= \left( \frac{1 \times 10 + 2 \times 4}{1+2}, \frac{1 \times (-16) + 2 \times 2}{1+2}, \frac{1 \times 6 + 2 \times (-6)}{1+2} \right) \\ &= \left( \frac{18}{3}, \frac{-12}{3}, \frac{-6}{3} \right) \\ &= (6, -4, -2)\end{aligned}$$

Similarly, we know that B divides the line segment PQ in the ratio 2: 1.

Upon comparing, we have

$$x_1 = 4, y_1 = 2, z_1 = -6;$$

$$x_2 = 10, y_2 = -16, z_2 = 6 \text{ and}$$

$$m = 2, n = 1$$

By using the section formula,

We know that the coordinates of the point R, which divide the line segment joining two points P ( $x_1, y_1, z_1$ ) and Q ( $x_2, y_2, z_2$ ) internally in the ratio m: n are given by

$$\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

So we have,

$$\begin{aligned}\text{The coordinates of B} &= \left( \frac{2 \times 10 + 1 \times 4}{2+1}, \frac{2 \times (-16) + 1 \times 2}{2+1}, \frac{2 \times 6 + 1 \times (-6)}{2+1} \right) \\ &= \left( \frac{24}{3}, \frac{-30}{3}, \frac{6}{3} \right) \\ &= (8, -10, 2)\end{aligned}$$

∴ The coordinates of the points which trisect the line segment joining the points P (4, 2, – 6) and Q (10, –16, 6) are (6, -4, -2) and (8, -10, 2).

Access Other Exercise Solutions of Class 11 Maths Chapter 12 – Introduction to Three Dimensional Geometry

Exercise 12.1 Solutions: 4 Questions

Exercise 12.2 Solutions: 5 Questions

Miscellaneous Exercise on Chapter 12 Solutions: 6 Questions

Also explore – NCERT Class 11 Solutions

