

Chapter 12: Introduction to Three Dimensional Geometry

Exercise 12.2

NCERT · Class 11 · Maths

Questions

Question 1

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1)

(ii) (-3, 7, 2) and (2, 4, -1)

(iii) (-1, 3, -4) and (1, -3, 4)

(iv) (2, -1, 3) and (-2, 1, 3)

Solution

(i) (2, 3, 5) and (4, 3, 1)

Let P be (2, 3, 5) and Q be (4, 3, 1)

By using the formula,

$$\text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 2, y_1 = 3, z_1 = 5$$

$$x_2 = 4, y_2 = 3, z_2 = 1$$

$$\text{Distance PQ} = \sqrt{(4 - 2)^2 + (3 - 3)^2 + (1 - 5)^2}$$

$$= \sqrt{(2)^2 + 0^2 + (-4)^2}$$

$$= \sqrt{4 + 0 + 16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

∴ The required distance is $2\sqrt{5}$ units.

(ii) $(-3, 7, 2)$ and $(2, 4, -1)$

Let P be $(-3, 7, 2)$, and Q be $(2, 4, -1)$

By using the formula,

$$\text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = -3, y_1 = 7, z_1 = 2$$

$$x_2 = 2, y_2 = 4, z_2 = -1$$

$$\text{Distance PQ} = \sqrt{(2 - (-3))^2 + (4 - 7)^2 + (-1 - 2)^2}$$

$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$

$$= \sqrt{25 + 9 + 9}$$

$$= \sqrt{43}$$

∴ The required distance is $\sqrt{43}$ units.

(iii) $(-1, 3, -4)$ and $(1, -3, 4)$

Let P be $(-1, 3, -4)$, and Q be $(1, -3, 4)$

By using the formula,

$$\text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = -1, y_1 = 3, z_1 = -4$$

$$x_2 = 1, y_2 = -3, z_2 = 4$$

$$\text{Distance PQ} = \sqrt{(1 - (-1))^2 + (-3 - 3)^2 + (4 - (-4))^2}$$

$$= \sqrt{(2)^2 + (-6)^2 + (8)^2}$$

$$= \sqrt{4 + 36 + 64}$$

$$= \sqrt{104}$$

$$= 2\sqrt{26}$$

∴ The required distance is $2\sqrt{26}$ units.

(iv) $(2, -1, 3)$ and $(-2, 1, 3)$

Let P be $(2, -1, 3)$, and Q be $(-2, 1, 3)$

By using the formula,

$$\text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 2, y_1 = -1, z_1 = 3$$

$$x_2 = -2, y_2 = 1, z_2 = 3$$

$$\text{Distance PQ} = \sqrt{(-2 - 2)^2 + (1 - (-1))^2 + (3 - 3)^2}$$

$$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$$

$$= \sqrt{16 + 4 + 0}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

∴ The required distance is $2\sqrt{5}$ units.

Question 2

Show that the points $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.

Solution

If three points are collinear, then they lie on the same line.

First, let us calculate the distance between the 3 points

i.e., PQ, QR and PR

Calculating PQ

$$P \equiv (-2, 3, 5) \text{ and } Q \equiv (1, 2, 3)$$

By using the formula,

$$\text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = -2, y_1 = 3, z_1 = 5$$

$$x_2 = 1, y_2 = 2, z_2 = 3$$

$$\text{Distance PQ} = \sqrt{[(1 - (-2))]^2 + (2 - 3)^2 + (3 - 5)^2}$$

$$= \sqrt{[(3)^2 + (-1)^2 + (-2)^2]}$$

$$= \sqrt{9 + 1 + 4}$$

$$= \sqrt{14}$$

Calculating QR

$$Q \equiv (1, 2, 3), \text{ and } R \equiv (7, 0, -1)$$

By using the formula,

$$\text{Distance QR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 1, y_1 = 2, z_1 = 3$$

$$x_2 = 7, y_2 = 0, z_2 = -1$$

$$\text{Distance QR} = \sqrt{[(7 - 1)^2 + (0 - 2)^2 + (-1 - 3)^2]}$$

$$= \sqrt{[(6)^2 + (-2)^2 + (-4)^2]}$$

$$= \sqrt{36 + 4 + 16}$$

$$= \sqrt{56}$$

$$= 2\sqrt{14}$$

Calculating PR

$$P \equiv (-2, 3, 5) \text{ and } R \equiv (7, 0, -1)$$

By using the formula,

$$\text{Distance PR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = -2, y_1 = 3, z_1 = 5$$

$$x_2 = 7, y_2 = 0, z_2 = -1$$

$$\text{Distance PR} = \sqrt{(7 - (-2))^2 + (0 - 3)^2 + (-1 - 5)^2}$$

$$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$$

$$= \sqrt{81 + 9 + 36}$$

$$= \sqrt{126}$$

$$= 3\sqrt{14}$$

$$\text{Thus, PQ} = \sqrt{14}, \text{QR} = 2\sqrt{14}, \text{and PR} = 3\sqrt{14}$$

$$\text{So, PQ} + \text{QR} = \sqrt{14} + 2\sqrt{14}$$

$$= 3\sqrt{14}$$

$$= \text{PR}$$

∴ The points P, Q and R are collinear.

Question 3

Verify the following:

(i) (0, 7, -10), (1, 6, -6), and (4, 9, -6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (-1, 6, 6), and (-4, 9, 6) are the vertices of a right-angled triangle.

(iii) (-1, 2, 1), (1, -2, 5), (4, -7, 8), and (2, -3, 4) are the vertices of a parallelogram.

Solution

(i) (0, 7, -10), (1, 6, -6), and (4, 9, -6) are the vertices of an isosceles triangle.

Let us consider the points

$$P(0, 7, -10), Q(1, 6, -6) \text{ and } R(4, 9, -6)$$

If any 2 sides are equal, it will be an isosceles triangle

So, first, let us calculate the distance of PQ, QR

Calculating PQ

$$P \equiv (0, 7, -10) \text{ and } Q \equiv (1, 6, -6)$$

By using the formula,

$$\text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 0, y_1 = 7, z_1 = -10$$

$$x_2 = 1, y_2 = 6, z_2 = -6$$

$$\text{Distance PQ} = \sqrt{(1 - 0)^2 + (6 - 7)^2 + (-6 - (-10))^2}$$

$$= \sqrt{(1)^2 + (-1)^2 + (4)^2}$$

$$= \sqrt{1 + 1 + 16}$$

$$= \sqrt{18}$$

Calculating QR

$$Q \equiv (1, 6, -6) \text{ and } R \equiv (4, 9, -6)$$

By using the formula,

$$\text{Distance QR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 1, y_1 = 6, z_1 = -6$$

$$x_2 = 4, y_2 = 9, z_2 = -6$$

$$\text{Distance QR} = \sqrt{(4 - 1)^2 + (9 - 6)^2 + (-6 - (-6))^2}$$

$$= \sqrt{(3)^2 + (3)^2 + (-6+6)^2}$$

$$= \sqrt{9 + 9 + 0}$$

$$= \sqrt{18}$$

Hence, PQ = QR

$$18 = 18$$

2 sides are equal

∴ PQR is an isosceles triangle.

(ii) (0, 7, 10), (−1, 6, 6), and (−4, 9, 6) are the vertices of a right-angled triangle.

Let the points be

P(0, 7, 10), Q(−1, 6, 6) & R(−4, 9, 6)

First, let us calculate the distance of PQ, OR and PR

Calculating PQ

P ≡ (0, 7, 10), and Q ≡ (−1, 6, 6)

By using the formula,

$$\text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 0, y_1 = 7, z_1 = 10$$

$$x_2 = -1, y_2 = 6, z_2 = 6$$

$$\text{Distance PQ} = \sqrt{(-1 - 0)^2 + (6 - 7)^2 + (6 - 10)^2}$$

$$= \sqrt{(-1)^2 + (-1)^2 + (-4)^2}$$

$$= \sqrt{1 + 1 + 16}$$

$$= \sqrt{18}$$

Calculating QR

Q ≡ (1, 6, −6), and R ≡ (4, 9, −6)

By using the formula,

$$\text{Distance QR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 1, y_1 = 6, z_1 = -6$$

$$x_2 = 4, y_2 = 9, z_2 = -6$$

$$\text{Distance QR} = \sqrt{[(4 - 1)^2 + (9 - 6)^2 + (-6 - (-6))^2]}$$

$$= \sqrt{[(3)^2 + (3)^2 + (-6+6)^2]}$$

$$= \sqrt{[9 + 9 + 0]}$$

$$= \sqrt{18}$$

Calculating PR

$$P \equiv (0, 7, 10) \text{ and } R \equiv (-4, 9, 6)$$

By using the formula,

$$\text{Distance PR} = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

So, here

$$x_1 = 0, y_1 = 7, z_1 = 10$$

$$x_2 = -4, y_2 = 9, z_2 = 6$$

$$\text{Distance PR} = \sqrt{[(-4 - 0)^2 + (9 - 7)^2 + (6 - 10)^2]}$$

$$= \sqrt{[(-4)^2 + (2)^2 + (-4)^2]}$$

$$= \sqrt{[16 + 4 + 16]}$$

$$= \sqrt{36}$$

Now,

$$PQ^2 + QR^2 = 18 + 18$$

$$= 36$$

$$= PR^2$$

By using the converse of Pythagoras theorem,

The given vertices P, Q & R are the vertices of a right-angled triangle at Q.

(iii) $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$, and $(2, -3, 4)$ are the vertices of a parallelogram.

Let the points be A $(-1, 2, 1)$, B $(1, -2, 5)$, C $(4, -7, 8)$, and D $(2, -3, 4)$

ABCD can be vertices of parallelogram only if opposite sides are equal.

i.e., $AB = CD$, and $BC = AD$

First, let us calculate the distance

Calculating AB

$A \equiv (-1, 2, 1)$ and $B \equiv (1, -2, 5)$

By using the formula,

$$\text{Distance AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = -1, y_1 = 2, z_1 = 1$$

$$x_2 = 1, y_2 = -2, z_2 = 5$$

$$\text{Distance AB} = \sqrt{[1 - (-1)]^2 + (-2 - 2)^2 + (5 - 1)^2}$$

$$= \sqrt{[2]^2 + (-4)^2 + (4)^2}$$

$$= \sqrt{4 + 16 + 16}$$

$$= \sqrt{36}$$

$$= 6$$

Calculating BC

$B \equiv (1, -2, 5)$ and $C \equiv (4, -7, 8)$

By using the formula,

$$\text{Distance BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 1, y_1 = -2, z_1 = 5$$

$$x_2 = 4, y_2 = -7, z_2 = 8$$

$$\text{Distance BC} = \sqrt{[4 - 1]^2 + (-7 - (-2))^2 + (8 - 5)^2}$$

$$= \sqrt{[3]^2 + (-5)^2 + (3)^2}$$

$$= \sqrt{9 + 25 + 9}$$

$$= \sqrt{43}$$

Calculating CD

$$C \equiv (4, -7, 8) \text{ and } D \equiv (2, -3, 4)$$

By using the formula,

$$\text{Distance CD} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 4, y_1 = -7, z_1 = 8$$

$$x_2 = 2, y_2 = -3, z_2 = 4$$

$$\text{Distance CD} = \sqrt{(2 - 4)^2 + (-3 - (-7))^2 + (4 - 8)^2}$$

$$= \sqrt{(-2)^2 + (4)^2 + (-4)^2}$$

$$= \sqrt{4 + 16 + 16}$$

$$= \sqrt{36}$$

$$= 6$$

Calculating DA

$$D \equiv (2, -3, 4) \text{ and } A \equiv (-1, 2, 1)$$

By using the formula,

$$\text{Distance DA} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = 2, y_1 = -3, z_1 = 4$$

$$x_2 = -1, y_2 = 2, z_2 = 1$$

$$\text{Distance DA} = \sqrt{(-1 - 2)^2 + (2 - (-3))^2 + (1 - 4)^2}$$

$$= \sqrt{(-3)^2 + (5)^2 + (-3)^2}$$

$$= \sqrt{9 + 25 + 9}$$

$$= \sqrt{43}$$

Since $AB = CD$ and $BC = DA$ (given),

In ABCD, both pairs of opposite sides are equal.

$\therefore$ ABCD is a parallelogram.

Question 4

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, -1).

Solution

Let A (1, 2, 3) & B (3, 2, -1)

Let point P be (x, y, z)

Since it is given that point P(x, y, z) is equal distance from point A(1, 2, 3) and B(3, 2, -1),

i.e., $PA = PB$

First, let us calculate

Calculating PA

$P \equiv (x, y, z)$ and $A \equiv (1, 2, 3)$

By using the formula,

$$\text{Distance PA} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = 1, y_2 = 2, z_2 = 3$$

$$\text{Distance PA} = \sqrt{(1 - x)^2 + (2 - y)^2 + (3 - z)^2}$$

Calculating PB

$P \equiv (x, y, z)$ and $B \equiv (3, 2, -1)$

By using the formula,

$$\text{Distance PB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = 3, y_2 = 2, z_2 = -1$$

$$\text{Distance PB} = \sqrt{(3 - x)^2 + (2 - y)^2 + (-1 - z)^2}$$

Since $PA = PB$,

Square on both sides, we get

$$PA^2 = PB^2$$

$$(1 - x)^2 + (2 - y)^2 + (3 - z)^2 = (3 - x)^2 + (2 - y)^2 + (-1 - z)^2$$

$$(1 + x^2 - 2x) + (4 + y^2 - 4y) + (9 + z^2 - 6z)$$

$$(9 + x^2 - 6x) + (4 + y^2 - 4y) + (1 + z^2 + 2z)$$

$$-2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$$

$$4x - 8z = 0$$

$$x - 2z = 0$$

∴ The required equation is $x - 2z = 0$

Question 5

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (-4, 0, 0) is equal to 10.

Solution

Let A (4, 0, 0), and B (-4, 0, 0)

Let the coordinates of point P be (x, y, z)

Calculating PA

$$P \equiv (x, y, z) \text{ and } A \equiv (4, 0, 0)$$

By using the formula,

$$\text{Distance PA} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = 4, y_2 = 0, z_2 = 0$$

$$\text{Distance PA} = \sqrt{(4 - x)^2 + (0 - y)^2 + (0 - z)^2}$$

Calculating PB

$$P \equiv (x, y, z) \text{ and } B \equiv (-4, 0, 0)$$

By using the formula,

$$\text{Distance PB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So, here

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = -4, y_2 = 0, z_2 = 0$$

$$\text{Distance PB} = \sqrt{(-4 - x)^2 + (0 - y)^2 + (0 - z)^2}$$

It is given that,

$$PA + PB = 10$$

$$PA = 10 - PB$$

Square on both sides, we get

$$PA^2 = (10 - PB)^2$$

$$PA^2 = 100 + PB^2 - 20 PB$$

$$(4 - x)^2 + (0 - y)^2 + (0 - z)^2$$

$$100 + (-4 - x)^2 + (0 - y)^2 + (0 - z)^2 - 20 PB$$

$$(16 + x^2 - 8x) + (y^2) + (z^2)$$

$$100 + (16 + x^2 + 8x) + (y^2) + (z^2) - 20 PB$$

$$20 PB = 16x + 100$$

$$5 PB = (4x + 25)$$

Square on both sides again, we get

$$25 PB^2 = 16x^2 + 200x + 625$$

$$25 [(-4 - x)^2 + (0 - y)^2 + (0 - z)^2] = 16x^2 + 200x + 625$$

$$25 [x^2 + y^2 + z^2 + 8x + 16] = 16x^2 + 200x + 625$$

$$25x^2 + 25y^2 + 25z^2 + 200x + 400 = 16x^2 + 200x + 625$$

$$9x^2 + 25y^2 + 25z^2 - 225 = 0$$

∴ The required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

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