

Chapter 12: Introduction To 3 D Geometry Miscellaneous Exercise

NCERT · Class 11 · Maths

Questions

Question 1

Three vertices of a parallelogram ABCD are A(3, -1, 2), B (1, 2, -4) and C (-1, 1, 2). Find the coordinates of the fourth vertex.

Solution

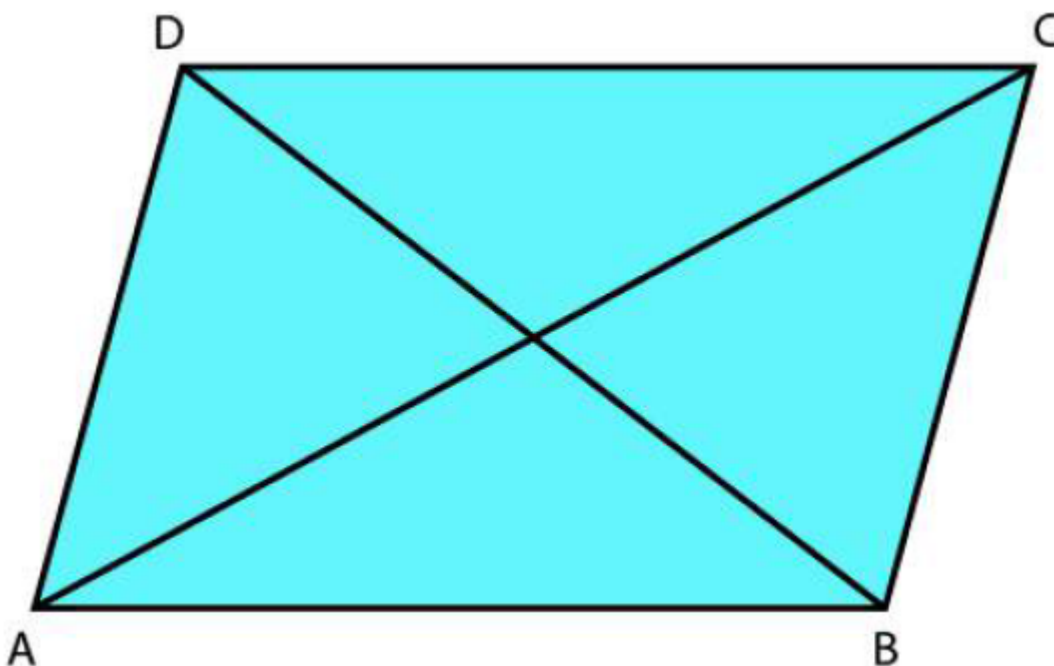
Given:

ABCD is a parallelogram with vertices A (3, -1, 2), B (1, 2, -4), C (-1, 1, 2).

Where, $x_1 = 3$, $y_1 = -1$, $z_1 = 2$

$x_2 = 1$, $y_2 = 2$, $z_2 = -4$

$x_3 = -1$, $y_3 = 1$, $z_3 = 2$



Let the coordinates of the fourth vertex be D (x, y, z).

We also know that the diagonals of a parallelogram bisect each other, so the midpoints of AC and BD are equal, i.e., Midpoint of AC = Midpoint of BD(1)

Now, by the Midpoint Formula, we know that the coordinates of the midpoint of the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are $[(x_1+x_2)/2, (y_1+y_2)/2, (z_1+z_2)/2]$

So we have,

Co-ordinates of the midpoint of AC:

$$= \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right)$$

$$= (2/2, 0/2, 4/2)$$

$$= (1, 0, 2)$$

$$1 + x = 2, 2 + y = 0, -4 + z = 4$$

$$x = 1, y = -2, z = 8$$

Hence, the coordinates of the fourth vertex are D (1, -2, 8).

Question 2

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Solution

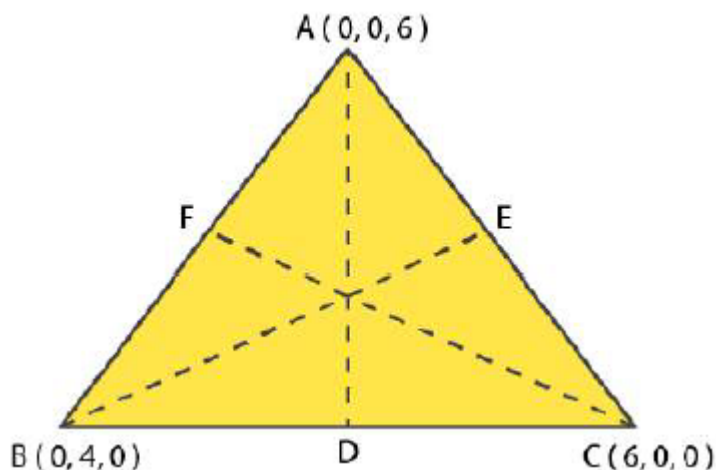
Given:

The vertices of the triangle are A (0, 0, 6), B (0, 4, 0) and C (6, 0, 0).

$$x_1 = 0, y_1 = 0, z_1 = 6$$

$$x_2 = 0, y_2 = 4, z_2 = 0$$

$$x_3 = 6, y_3 = 0, z_3 = 0$$



So, let the medians of this triangle be AD, BE and CF, corresponding to the vertices A, B and C, respectively.

D, E and F are the midpoints of the sides BC, AC and AB, respectively.

By the Midpoint Formula, we know that the coordinates of the mid-point of the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are $[(x_1+x_2)/2, (y_1+y_2)/2, (z_1+z_2)/2]$

So we have,

The coordinates of D:

$$= \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) = \left(\frac{6}{2}, \frac{4}{2}, \frac{0}{2} \right) \\ = (3, 2, 0)$$

The coordinates of E:

$$= \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2} \right) = \left(\frac{6}{2}, \frac{0}{2}, \frac{6}{2} \right) \\ = (3, 0, 3)$$

And the coordinates of F:

$$= \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2} \right) = \left(\frac{0}{2}, \frac{4}{2}, \frac{6}{2} \right) \\ = (0, 2, 3)$$

By the Distance Formula, we know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So the lengths of the medians are:

$$\begin{aligned} AD &= \sqrt{(3 - 0)^2 + (2 - 0)^2 + (0 - 6)^2} = \sqrt{3^2 + 2^2 + (-6)^2} = \sqrt{9 + 4 + 36} \\ &= \sqrt{49} = 7 \end{aligned}$$

$$\begin{aligned} BE &= \sqrt{(3 - 0)^2 + (0 - 4)^2 + (3 - 0)^2} = \sqrt{3^2 + (-4)^2 + 3^2} = \sqrt{9 + 16 + 9} \\ &= \sqrt{34} \end{aligned}$$

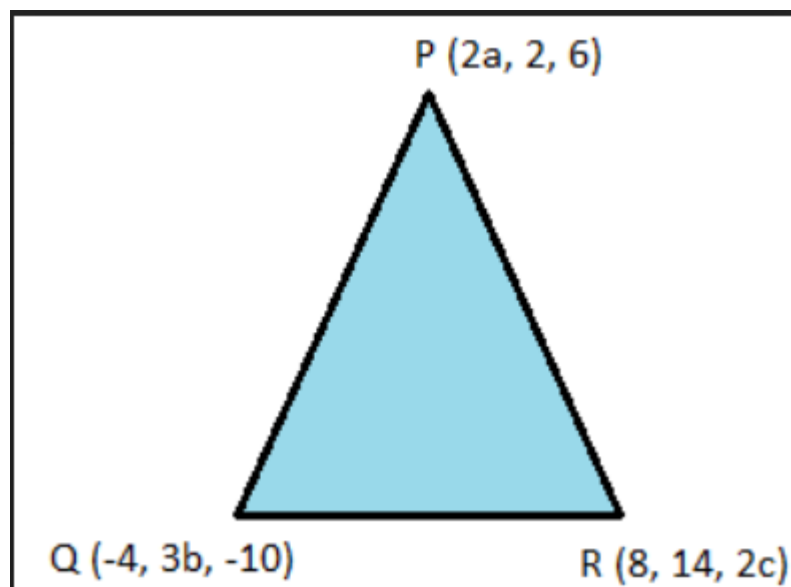
$$\begin{aligned} CF &= \sqrt{(0 - 6)^2 + (2 - 0)^2 + (3 - 0)^2} = \sqrt{(-6)^2 + 2^2 + 3^2} = \sqrt{36 + 4 + 9} \\ &= \sqrt{49} = 7 \end{aligned}$$

∴ The lengths of the medians of the given triangle are 7, $\sqrt{34}$ and 7.

Question 3

If the origin is the centroid of the triangle PQR with vertices P (2a, 2, 6), Q (-4, 3b, -10) and R(8, 14, 2c), then find the values of a, b and c.

Solution



Given:

The vertices of the triangle are P (2a, 2, 6), Q (-4, 3b, -10) and R (8, 14, 2c).

Where,

$$x_1 = 2a, y_1 = 2, z_1 = 6;$$

$$x_2 = -4, y_2 = 3b, z_2 = -10;$$

$$x_3 = 8, y_3 = 14, z_3 = 2c$$

We know that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) , are $[(x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3, (z_1+z_2+z_3)/3]$

So, the coordinates of the centroid of the triangle PQR are

$$\left(\frac{2a - 4 + 8}{3}, \frac{2 + 3b + 14}{3}, \frac{6 - 10 + 2c}{3} \right) = \left(\frac{2a + 4}{3}, \frac{3b + 16}{3}, \frac{2c - 4}{3} \right)$$

Now, it is given that the origin $(0, 0, 0)$ is the centroid.

$$\text{So, we have } \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3} \right) = (0, 0, 0)$$

$$\frac{2a + 4}{3} = 0, \frac{3b + 16}{3} = 0, \frac{2c - 4}{3} = 0$$

$$2a + 4 = 0, 3b + 16 = 0, 2c - 4 = 0$$

$$a = -2, b = -16/3, c = 2$$

$\therefore$ The values of a, b and c are $a = -2, b = -16/3, c = 2$

Question 4

Find the coordinates of a point on the y-axis, which are at a distance of $5\sqrt{2}$ from the point P $(3, -2, 5)$.

Solution

Let the point on the y-axis be A $(0, y, 0)$.

Then, it is given that the distance between the points A $(0, y, 0)$ and P $(3, -2, 5)$ is $5\sqrt{2}$.

Now, by using the distance formula,

We know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$$\text{Distance of PQ} = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

So, the distance between the points A $(0, y, 0)$ and P $(3, -2, 5)$ is given by

$$\text{Distance of AP} = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

$$= \sqrt{[(3-0)^2 + (-2-y)^2 + (5-0)^2]}$$

$$= \sqrt{[3^2 + (-2-y)^2 + 5^2]}$$

$$= \sqrt{[(-2-y)^2 + 9 + 25]}$$

$$5\sqrt{2} = \sqrt{[(-2-y)^2 + 34]}$$

Squaring on both sides, we get

$$(-2-y)^2 + 34 = 25 \times 2$$

$$(-2-y)^2 = 50 - 34$$

$$4 + y^2 + (2 \times -2 \times -y) = 16$$

$$y^2 + 4y + 4 - 16 = 0$$

$$y^2 + 4y - 12 = 0$$

$$y^2 + 6y - 2y - 12 = 0$$

$$y(y+6) - 2(y+6) = 0$$

$$(y+6)(y-2) = 0$$

$$y = -6, y = 2$$

$\therefore$ The points (0, 2, 0) and (0, -6, 0) are the required points on the y-axis.

Question 5

A point R with x-coordinate 4 lies on the line segment joining the points P (2, -3, 4) and Q (8, 0, 10).

Find the coordinates of the point R.

[Hint: Suppose R divides PQ in the ratio k: 1. The coordinates of the point R are given by

$$\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)]$$

Solution

Given:

The coordinates of the points P (2, -3, 4) and Q (8, 0, 10).

$$x_1 = 2, y_1 = -3, z_1 = 4$$

$$x_2 = 8, y_2 = 0, z_2 = 10$$

Let the coordinates of the required point be $(4, y, z)$.

So now, let the point R $(4, y, z)$ divides the line segment joining the points P $(2, -3, 4)$ and Q $(8, 0, 10)$ in the ratio k: 1.

By using the Section Formula,

We know that the coordinates of the point R, which divides the line segment joining two points, P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) , internally in the ratio m: n, are given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

So, the coordinates of the point R are given by

$$\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)$$

So, we have

$$\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right) = (4, y, z)$$

$$\Rightarrow \frac{8k+2}{k+1} = 4$$

$$8k + 2 = 4(k + 1)$$

$$8k + 2 = 4k + 4$$

$$8k - 4k = 4 - 2$$

$$4k = 2$$

$$k = 2/4$$

$$= 1/2$$

Now, let us substitute the values, and we get

$$\Rightarrow y = \frac{-3}{\frac{1}{2} + 1} = \frac{-3}{\frac{3}{2}} = \frac{-3 \times 2}{3} = -2,$$

$$z = \frac{10\left(\frac{1}{2}\right) + 4}{\frac{1}{2} + 1} = \frac{5 + 4}{\frac{3}{2}} = \frac{9 \times 2}{3} = 3 \times 2$$

$$= 6$$

$\therefore$ The coordinates of the required point are (4, -2, 6).

Question 6

If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Solution

Given:

Points A (3, 4, 5) and B (-1, 3, -7)

$$x_1 = 3, y_1 = 4, z_1 = 5$$

$$x_2 = -1, y_2 = 3, z_2 = -7$$

$$PA^2 + PB^2 = k^2 \dots\dots\dots(1)$$

Let the point be P (x, y, z).

Now by using the distance formula,

We know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So,

$$PA = \sqrt{(3 - x)^2 + (4 - y)^2 + (5 - z)^2}$$

And

$$PB = \sqrt{(-1 - x)^2 + (3 - y)^2 + (-7 - z)^2}$$

Now, substituting these values in (1), we have

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$$9 + x^2 - 6x + 16 + y^2 - 8y + 25 + z^2 - 10z + 1 + x^2 + 2x + 9 + y^2 - 6y + 49 + z^2 + 14z = k^2$$

$$2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$

$$2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = k^2 - 109$$

$$2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$$

$$(x^2 + y^2 + z^2 - 2x - 7y + 2z) = (k^2 - 109)/2$$

Hence, the required equation is $(x^2 + y^2 + z^2 - 2x - 7y + 2z) = (k^2 - 109)/2$

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