

Chapter 11: Introduction to Three Dimensional Geometry

NCERT · Class 11 · Maths

Exercise 11.1

Question 1

A point is on the x -axis. What are its y -coordinate and z -coordinates?

Solution

Ans: We know that, the coordinates of any point on the x -axis will be as $(x, 0, 0)$.

Thus, if a point is on the x -axis, then its y -coordinate will be 0 and z -coordinate will be 0.

Question 2

A point is in the XZ -plane. What can you say about its y -coordinate?

Solution

Ans: We know that, the coordinates of any point in the XZ -plane will be as $(x, 0, z)$.

So, in the XZ - plane $y = 0$.

Thus, if a point is in the XZ - plane then the coordinate of y will always be 0.

Hence, its y -coordinate is 0.

Question 3

Name the octants in which the following points lie:

$(1, 2, 3)$

Solution

Ans: We have the following table,

Octant	Coordinates	I	II	III	IV	V	VI	VII	VIII

x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

From the above table, the point $(1, 2, 3)$ lies in the first octant.

$(4, -2, 3)$

Ans: We have the following table,

Octant Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

From the above table, the point $(4, -2, 3)$ lies in octant IV.

$(4, -2, -5)$

Ans: We have the following table,

Octant Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

From the above table, the point $(4, -2, -5)$ lies in octant VIII.

$(4, 2, -5)$

Ans: We have the following table,

From the above table, the point $(4, 2, -5)$ lies in octant V.

$(-4, 2, -5)$

Ans: We have the following table,

Octant Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

From the above table, the point $(-4, 2, -5)$ lies in octant VI.

$(-4, 2, 5)$

Ans: We have the following table,

Octant Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

From the above table, the point $(-4, 2, 5)$ lies in octant II.

$(-3, -1, 6)$

Ans: We have the following table,

Octant Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

From the above table, the point $(-3, -1, 6)$ lies in octant III.

$(-2, -4, -7)$

Ans: We have the following table,

Octant Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

From the above table, the point $(-2, -4, -7)$ lies in octant VII.

Question 4

Fill in the blanks:

- (i) The x -axis and y -axis taken together determine a plane known as _____ .
- (ii) The coordinates of points in the XY -plane are of the form _____ .
- (iii) Coordinate planes divide the space into _____ octants.

Solution

Ans: In XY -plane, $z = 0$.

Hence, the x -axis and y -axis taken together determine a plane known as XY -plane.

Ans: In XY -plane, $z = 0$.

Let x -coordinate be x and y -coordinate be y .

Hence, the coordinates of points in the XY -plane are of the form $(x, y, 0)$.

Ans: Coordinate planes divide the space into 8 octants.

Exercise 11.2

Question 1

Find the distance between the following pairs of points:

1. $(2, 3, 5)$ and $(4, 3, 1)$
2. $(-3, 7, 2)$ and $(2, 4, -1)$
3. $(-1, 3, -4)$ and $(1, -3, 4)$
4. $(2, -1, 3)$ and $(-2, 1, 3)$

Solution

Ans: (i) $(2, 3, 5)$ and $(4, 3, 1)$:

We use the distance formula between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$d = \sqrt{(4 - 2)^2 + (3 - 3)^2 + (1 - 5)^2} = \sqrt{(2)^2 + (0)^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

(ii) $(-3, 7, 2)$ and $(2, 4, -1)$:

$$d = \sqrt{(2 + 3)^2 + (4 - 7)^2 + (-1 - 2)^2} = \sqrt{(5)^2 + (-3)^2 + (-3)^2} = \sqrt{25 + 9 + 9} = \sqrt{43}$$

(iii) $(-1, 3, -4)$ and $(1, -3, 4)$:

$$d = \sqrt{(1 + 1)^2 + (-3 - 3)^2 + (4 + 4)^2} = \sqrt{(2)^2 + (-6)^2 + (8)^2} = \sqrt{4 + 36 + 64} = \sqrt{104} = 2\sqrt{26}$$

(iv) $(2, -1, 3)$ and $(-2, 1, 3)$:

$$d = \sqrt{(-2 - 2)^2 + (1 + 1)^2 + (3 - 3)^2} = \sqrt{(-4)^2 + (2)^2 + (0)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$

Question 2

Show that the points $(-2, 3, 5)$, $(1, 2, 3)$, and $(7, 0, -1)$ are collinear.

Solution

Ans: To prove collinearity, we check if the distances between the points satisfy $AB + BC = AC$.

- Distance between $A(-2, 3, 5)$ and $B(1, 2, 3)$:

$$AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{(3)^2 + (-1)^2 + (-2)^2} = \sqrt{9+1+4} = \sqrt{14}$$

- Distance between $B(1, 2, 3)$ and $C(7, 0, -1)$:

$$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{(6)^2 + (-2)^2 + (-4)^2} = \sqrt{36+4+16} = \sqrt{56}$$

- Distance between $A(-2, 3, 5)$ and $C(7, 0, -1)$:

$$AC = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} = \sqrt{(9)^2 + (-3)^2 + (-6)^2} = \sqrt{81+9+36} = \sqrt{126}$$

Since $AB + BC \neq AC$, the points are not collinear.

Question 3

Verify the following:

(i) Show that $(0, 7, -10)$, $(1, 6, -6)$, and $(4, 9, -6)$ are vertices of an isosceles triangle.

(ii) Show that $(0, 7, 10)$, $(-1, 6, 6)$, and $(4, 9, 6)$ are the vertices of a right-angled triangle.

(iii) Show that $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$, and $(2, -3, 4)$ are the vertices of a parallelogram.

Solution

Ans:

We check if two sides of the triangle are equal:

- Distance between $(0, 7, -10)$ and $(1, 6, -6)$:

$$d = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} = \sqrt{1+1+16} = \sqrt{18}$$

- Distance between $(1, 6, -6)$ and $(4, 9, -6)$:

$$d = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} = \sqrt{9+9+0} = \sqrt{18}$$

- Distance between $(0, 7, -10)$ and $(4, 9, -6)$:

$$d = \sqrt{(4-0)^2 + (9-7)^2 + (-6+10)^2} = \sqrt{16+4+16} = \sqrt{36}$$

Since two sides are equal, the triangle is isosceles.

Ans: To verify if the triangle is right-angled, we check if the square of the longest side equals the sum of the squares of the other two sides.

- Distance between $(0, 7, 10)$ and $(-1, 6, 6)$:

$$d_1 = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} = \sqrt{(-1)^2 + (-1)^2 + (-4)^2} = \sqrt{1+1+16} = \sqrt{18}$$

- Distance between $(-1, 6, 6)$ and $(4, 9, 6)$:

$$d_2 = \sqrt{(4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{(5)^2 + (3)^2 + (0)^2} = \sqrt{25+9+0} = \sqrt{34}$$

- Distance between $(0, 7, 10)$ and $(4, 9, 6)$:

$$d_3 = \sqrt{(4-0)^2 + (9-7)^2 + (6-10)^2} = \sqrt{(4)^2 + (2)^2 + (-4)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$$

Now check if the Pythagorean theorem holds:

$$d_1^2 + d_2^2 = 18 + 34 = 52$$

$$d_3^2 = 6^2 = 36$$

Since $d_1^2 + d_2^2 \neq d_3^2$, the triangle is not right-angled based on this configuration.

Ans: To verify if the points form a parallelogram, we need to show that the diagonals bisect each other, i.e., the midpoints of both diagonals should coincide.

- Midpoint of diagonal $(-1, 2, 1)$ and $(4, -7, 8)$:

$$M_1 = \left(\frac{-1+4}{2}, \frac{2-7}{2}, \frac{1+8}{2} \right) = \left(\frac{3}{2}, \frac{-5}{2}, \frac{9}{2} \right)$$

- Midpoint of diagonal $(1, -2, 5)$ and $(2, -3, 4)$:

$$M_2 = \left(\frac{1+2}{2}, \frac{-2-3}{2}, \frac{5+4}{2} \right) = \left(\frac{3}{2}, \frac{-5}{2}, \frac{9}{2} \right)$$

Since the midpoints are the same, the diagonals bisect each other, proving that the points form a parallelogram.

Question 4

Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$.

Solution

Ans: Let the coordinates of the point $P(x, y, z)$ be equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$. Then,

$$\text{Distance from } P(x, y, z) \text{ to } (1, 2, 3) = \text{Distance from } P(x, y, z) \text{ to } (3, 2, -1)$$

Using the distance formula:

$$\sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$$

Squaring both sides:

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

Expanding both sides:

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 - 6z + 9) = (x^2 - 6x + 9) + (y^2 - 4y + 4) + (z^2 + 2z + 1)$$

Simplifying:

$$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

Cancel the common terms:

$$-2x + 1 - 6z + 9 = -6x + 9 + 2z + 1$$

Simplifying further:

$$-2x - 6z + 10 = -6x + 2z + 10$$

Bringing all terms to one side:

$$4x - 8z = 0$$

Thus, the equation of the set of points is:

$$x = 2z$$

Question 5

Find the equation of the set of points P , the sum of whose distances from $A(4, 0, 0)$ and $B(-4, 0, 0)$ is equal to 10.

Solution

Ans: This is the equation of an ellipse with foci at points $A(4, 0, 0)$ and $B(-4, 0, 0)$, and the sum of the distances from these points is equal to the constant 10. The standard equation of an ellipse with foci at $(a, 0, 0)$ and $(-a, 0, 0)$, and the sum of the distances equal to $2c$, is:

$$2a = 10, \Rightarrow a = 5, \quad b^2 = a^2 - f^2$$

Here, the distance between the foci is 8, so $f = 4$. Thus:

$$b^2 = 5^2 - 4^2 = 25 - 16 = 9, \quad b = 3$$

Therefore, the equation of the ellipse is:

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Miscellaneous Exercise**Question 1**

Three vertices of a parallelogram ABCD are A (3,-1,2), B (1,2,-4) and C (-1,1,2). Find the coordinates of the fourth vertex.

Solution

Ans: We are given the three vertices of a parallelogram ABCD are A (3,-1,2), B (1,2,-4) and C (-1,1,2).

Let the coordinates of the fourth vertex of the parallelogram ABCD be D (x,y,z).

According to the property of a parallelogram, the diagonals of the parallelogram bisect each other.

In this parallelogram ABCD, AC and BD at point O.

So,

Mid-point of AC = Mid-point of BD

$$\left(\frac{3+(-1)}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$(1,0,2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$\frac{x+1}{2} = 1$$

$$\frac{y+2}{2} = 0$$

$$\frac{z-4}{2} = 2$$

We get, $x = 1$, $y = 2$ and $z = 8$

Therefore, the coordinates of the fourth vertex of the parallelogram ABCD are D (1,-2,8).

Question 2

Find the lengths of the medians of the triangle with A (0,0,6), B (0,4,0) and C (6,0,0).

Solution

Ans: For the given triangle ABC. Let AD, BE and CF are the medians:

We know that the median divides the line segment into two equal parts, so D is the midpoint of BC, therefore,

$$\text{Coordinates of point D} = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right)$$

$$\text{Coordinates of point D} = (3,2,0)$$

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2}$$

$$AD = \sqrt{9+4+36}$$

$$AD = \sqrt{49}$$

$$AD = 7$$

Similarly, E is the midpoint of AC,

$$\text{Coordinates of point E} = \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{0+6}{2} \right)$$

$$\text{Coordinates of point E} = (3,0,3)$$

$$AC = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2}$$

$$AC = \sqrt{9+16+9}$$

$$AC = \sqrt{34}$$

Similarly, F is the midpoint of AB,

$$\text{Coordinates of point F} = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2} \right)$$

$$\text{Coordinates of point F} = (0, 2, 3)$$

$$CF = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2}$$

$$CF = \sqrt{36+4+9}$$

$$CF = \sqrt{49}$$

$$CF = 7$$

Therefore, the lengths of the medians of the triangle ABC we obtain are, 7, $\sqrt{34}$, 7

Question 3

If the origin is the centroid of the triangle PQR with vertices P (2a, 2, 6), Q (-4, 3b, -10) and R (8, 14, 2c), then find the values of a, b and c

Solution

Ans: The given triangle PQR

We know that the coordinates of the centroid of triangle with the vertices (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) are,

$$\frac{x_1+x_2+x_3}{3} = \frac{y_1+y_2+y_3}{3} = \frac{z_1+z_2+z_3}{3}$$

For triangle PQR, the coordinates will be,

$$\Delta PQR = \frac{2a-4+8}{3} = \frac{2+3b+14}{3} = \frac{6-10+2c}{3}$$

$$\Delta PQR = \frac{2a+4}{3} = \frac{3b+16}{3} = \frac{2c-4}{3}$$

Now, we are given that centroid is the origin,

$$\frac{2a+4}{3} = 0$$

$$\frac{3b+16}{3} = 0$$

$$\frac{2c-4}{3} = 0$$

$$a = -2,$$

$$b = -\frac{16}{3}$$

$$c = 2$$

Therefore, we obtain the values as $a = -2$, $b = -\frac{16}{3}$ and $c = 2$.

Question 4

If A and B be the points (3,4,5) and (-1,3,-7) respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Solution

Ans: Let the coordinates of point P be (x,y,z).

Using the distance formula we get,

$$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$$

$$PA^2 = x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z$$

$$PA^2 = x^2 - 6x + y^2 - 8y + z^2 - 10z + 50$$

Similarly,

$$PB^2 = (x-1)^2 + (y-3)^2 + (z-7)^2$$

$$PB^2 = x^2 - 2x + y^2 - 6y + z^2 - 14z + 59$$

We are given that,

$$PA^2 + PB^2 = k^2$$

So,

$$(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 - 2x + y^2 - 6y + z^2 - 14z + 59) = k^2$$

$$2x^2 + 2y^2 + 2z^2 - 4x - 14y + 14z + 109 = k^2$$

$$2(x^2 + y^2 + z^2 - 2x - 7y + 7z) = k^2 - 109$$

$$x^2 + y^2 + z^2 - 2x - 7y + 7z = \frac{k^2 - 109}{2}$$

Therefore, the equation is as follows $x^2 + y^2 + z^2 - 2x - 7y + 7z = \frac{k^2 - 109}{2}$