

Chapter 11: Conic Sections Exercise 11.3

NCERT · Class 11 · Maths

Questions

Question 1

$$x^2/36 + y^2/16 = 1$$

Solution

Given:

The equation is $x^2/36 + y^2/16 = 1$

Here, the denominator of $x^2/36$ is greater than the denominator of $y^2/16$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$$a = 6 \text{ and } b = 4.$$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{36-16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

Then,

The coordinates of the foci are $(2\sqrt{5}, 0)$ and $(-2\sqrt{5}, 0)$.

The coordinates of the vertices are $(6, 0)$ and $(-6, 0)$

$$\text{Length of major axis} = 2a = 2(6) = 12$$

$$\text{Length of minor axis} = 2b = 2(4) = 8$$

$$\text{Eccentricity, } e = c/a = 2\sqrt{5}/6 = \sqrt{5}/3$$

$$\text{Length of latus rectum} = 2b^2/a = (2 \times 16)/6 = 16/3$$

Question 2

$$x^2/4 + y^2/25 = 1$$

Solution

Given:

$$\text{The equation is } x^2/4 + y^2/25 = 1$$

Here, the denominator of $y^2/25$ is greater than the denominator of $x^2/4$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$$a = 5 \text{ and } b = 2.$$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{25-4}$$

$$= \sqrt{21}$$

Then,

The coordinates of the foci are $(0, \sqrt{21})$ and $(0, -\sqrt{21})$.

The coordinates of the vertices are $(0, 5)$ and $(0, -5)$

$$\text{Length of major axis} = 2a = 2(5) = 10$$

$$\text{Length of minor axis} = 2b = 2(2) = 4$$

$$\text{Eccentricity, } e = c/a = \sqrt{21}/5$$

$$\text{Length of latus rectum} = 2b^2/a = (2 \times 2^2)/5 = (2 \times 4)/5 = 8/5$$

Question 3

$$x^2/16 + y^2/9 = 1$$

Solution

Given:

The equation is $x^2/16 + y^2/9 = 1$ or $x^2/4^2 + y^2/3^2 = 1$

Here, the denominator of $x^2/16$ is greater than the denominator of $y^2/9$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$a = 4$ and $b = 3$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{16-9}$$

$$= \sqrt{7}$$

Then,

The coordinates of the foci are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$.

The coordinates of the vertices are $(4, 0)$ and $(-4, 0)$

Length of major axis = $2a = 2(4) = 8$

Length of minor axis = $2b = 2(3) = 6$

Eccentricity, $e = c/a = \sqrt{7}/4$

Length of latus rectum = $2b^2/a = (2 \times 3^2)/4 = (2 \times 9)/4 = 18/4 = 9/2$

Question 4

$$x^2/25 + y^2/100 = 1$$

Solution

Given:

The equation is $x^2/25 + y^2/100 = 1$

Here, the denominator of $y^2/100$ is greater than the denominator of $x^2/25$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$$b = 5 \text{ and } a = 10.$$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{100 - 25}$$

$$= \sqrt{75}$$

$$= 5\sqrt{3}$$

Then,

The coordinates of the foci are $(0, 5\sqrt{3})$ and $(0, -5\sqrt{3})$.

The coordinates of the vertices are $(0, \sqrt{10})$ and $(0, -\sqrt{10})$

$$\text{Length of major axis} = 2a = 2(10) = 20$$

$$\text{Length of minor axis} = 2b = 2(5) = 10$$

$$\text{Eccentricity, } e = c/a = 5\sqrt{3}/10 = \sqrt{3}/2$$

$$\text{Length of latus rectum} = 2b^2/a = (2 \times 5^2)/10 = (2 \times 25)/10 = 5$$

Question 5

$$x^2/49 + y^2/36 = 1$$

Solution

Given:

$$\text{The equation is } x^2/49 + y^2/36 = 1$$

Here, the denominator of $x^2/49$ is greater than the denominator of $y^2/36$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$$b = 6 \text{ and } a = 7$$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{49-36}$$

$$= \sqrt{13}$$

Then,

The coordinates of the foci are $(\sqrt{13}, 0)$ and $(-\sqrt{13}, 0)$.

The coordinates of the vertices are $(7, 0)$ and $(-7, 0)$

$$\text{Length of major axis} = 2a = 2(7) = 14$$

$$\text{Length of minor axis} = 2b = 2(6) = 12$$

$$\text{Eccentricity, } e = c/a = \sqrt{13}/7$$

$$\text{Length of latus rectum} = 2b^2/a = (2 \times 6^2)/7 = (2 \times 36)/7 = 72/7$$

Question 6

$$x^2/100 + y^2/400 = 1$$

Solution

Given:

$$\text{The equation is } x^2/100 + y^2/400 = 1$$

Here, the denominator of $y^2/400$ is greater than the denominator of $x^2/100$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$$b = 10 \text{ and } a = 20.$$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{400-100}$$

$$= \sqrt{300}$$

$$= 10\sqrt{3}$$

Then,

The coordinates of the foci are $(0, 10\sqrt{3})$ and $(0, -10\sqrt{3})$.

The coordinates of the vertices are $(0, 20)$ and $(0, -20)$

Length of major axis = $2a = 2(20) = 40$

Length of minor axis = $2b = 2(10) = 20$

Eccentricity, $e = c/a = 10\sqrt{3}/20 = \sqrt{3}/2$

Length of latus rectum = $2b^2/a = (2 \times 10^2)/20 = (2 \times 100)/20 = 10$

Question 7

$$36x^2 + 4y^2 = 144$$

Solution

Given:

The equation is $36x^2 + 4y^2 = 144$ or $x^2/4 + y^2/36 = 1$ or $x^2/2^2 + y^2/6^2 = 1$

Here, the denominator of $y^2/6^2$ is greater than the denominator of $x^2/2^2$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$b = 2$ and $a = 6$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{36-4}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

Then,

The coordinates of the foci are $(0, 4\sqrt{2})$ and $(0, -4\sqrt{2})$.

The coordinates of the vertices are $(0, 6)$ and $(0, -6)$

Length of major axis = $2a = 2(6) = 12$

Length of minor axis = $2b = 2(2) = 4$

Eccentricity, $e = c/a = 4\sqrt{2}/6 = 2\sqrt{2}/3$

Length of latus rectum $= 2b^2/a = (2 \times 2^2)/6 = (2 \times 4)/6 = 4/3$

Question 8

$$16x^2 + y^2 = 16$$

Solution

Given:

The equation is $16x^2 + y^2 = 16$ or $x^2/1 + y^2/16 = 1$ or $x^2/1^2 + y^2/4^2 = 1$

Here, the denominator of $y^2/4^2$ is greater than the denominator of $x^2/1^2$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$b = 1$ and $a = 4$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{16 - 1}$$

$$= \sqrt{15}$$

Then,

The coordinates of the foci are $(0, \sqrt{15})$ and $(0, -\sqrt{15})$.

The coordinates of the vertices are $(0, 4)$ and $(0, -4)$

Length of major axis $= 2a = 2(4) = 8$

Length of minor axis $= 2b = 2(1) = 2$

Eccentricity, $e = c/a = \sqrt{15}/4$

Length of latus rectum $= 2b^2/a = (2 \times 1^2)/4 = 2/4 = \frac{1}{2}$

Question 9

$$4x^2 + 9y^2 = 36$$

Solution

Given:

The equation is $4x^2 + 9y^2 = 36$ or $x^2/9 + y^2/4 = 1$ or $x^2/3^2 + y^2/2^2 = 1$

Here, the denominator of $x^2/3^2$ is greater than the denominator of $y^2/2^2$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$a = 3$ and $b = 2$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{9-4}$$

$$= \sqrt{5}$$

Then,

The coordinates of the foci are $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.

The coordinates of the vertices are $(3, 0)$ and $(-3, 0)$

Length of major axis = $2a = 2(3) = 6$

Length of minor axis = $2b = 2(2) = 4$

Eccentricity, $e = c/a = \sqrt{5}/3$

Length of latus rectum = $2b^2/a = (2 \times 2^2)/3 = (2 \times 4)/3 = 8/3$

In each of the following Exercises 10 to 20, find the equation for the ellipse that satisfies the given conditions:

Question 10

Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$

Solution

Given:

Vertices $(\pm 5, 0)$ and foci $(\pm 4, 0)$

Here, the vertices are on the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 5$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\text{So, } 5^2 = b^2 + 4^2$$

$$25 = b^2 + 16$$

$$b^2 = 25 - 16$$

$$b = \sqrt{9}$$

$$= 3$$

$\therefore$ The equation of the ellipse is $x^2/5^2 + y^2/3^2 = 1$ or $x^2/25 + y^2/9 = 1$

Question 11

Vertices $(0, \pm 13)$, foci $(0, \pm 5)$

Solution

Given:

Vertices $(0, \pm 13)$ and foci $(0, \pm 5)$

Here, the vertices are on the y-axis.

So, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$$13^2 = b^2 + 5^2$$

$$169 = b^2 + 25$$

$$b^2 = 169 - 25$$

$$b = \sqrt{144}$$

$$= 12$$

∴ The equation of the ellipse is $x^2/12^2 + y^2/13^2 = 1$ or $x^2/144 + y^2/169 = 1$

Question 12

Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$

Solution

Given:

Vertices $(\pm 6, 0)$ and foci $(\pm 4, 0)$

Here, the vertices are on the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 6$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$6^2 = b^2 + 4^2$$

$$36 = b^2 + 16$$

$$b^2 = 36 - 16$$

$$b = \sqrt{20}$$

∴ The equation of the ellipse is $x^2/6^2 + y^2/(\sqrt{20})^2 = 1$ or $x^2/36 + y^2/20 = 1$

Question 13

Ends of major axis $(\pm 3, 0)$, ends of minor axis $(0, \pm 2)$

Solution

Given:

Ends of the major axis $(\pm 3, 0)$ and ends of minor axis $(0, \pm 2)$

Here, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 3$ and $b = 2$.

∴ The equation for the ellipse $x^2/3^2 + y^2/2^2 = 1$ or $x^2/9 + y^2/4 = 1$

Question 14

Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

Solution

Given:

Ends of major axis $(0, \pm\sqrt{5})$ and ends of minor axis $(\pm 1, 0)$

Here, the major axis is along the y-axis.

So, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

Then, $a = \sqrt{5}$ and $b = 1$.

∴ The equation for the ellipse $x^2/1^2 + y^2/(\sqrt{5})^2 = 1$ or $x^2/1 + y^2/5 = 1$

Question 15

Length of major axis 26, foci $(\pm 5, 0)$

Solution

Given:

Length of the major axis is 26 and the foci $(\pm 5, 0)$

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $2a = 26$

$a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$$13^2 = b^2 + 5^2$$

$$169 = b^2 + 25$$

$$b^2 = 169 - 25$$

$$b = \sqrt{144}$$

$$= 12$$

$\therefore$ The equation of the ellipse is $x^2/13^2 + y^2/12^2 = 1$ or $x^2/169 + y^2/144 = 1$

Question 16

Length of minor axis 16, foci (0, ± 6).

Solution

Given:

Length of minor axis is 16 and foci (0, ± 6).

Since the foci are on the y-axis, the major axis is along the y-axis.

So, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

Then, $2b = 16$

$b = 8$ and $c = 6$.

It is known that $a^2 = b^2 + c^2$.

$$a^2 = 8^2 + 6^2$$

$$= 64 + 36$$

$$= 100$$

$$a = \sqrt{100}$$

$$= 10$$

$\therefore$ The equation of the ellipse is $x^2/8^2 + y^2/10^2 = 1$ or $x^2/64 + y^2/100 = 1$

Question 17

Foci (± 3 , 0), a = 4

Solution

Given:

Foci $(\pm 3, 0)$ and $a = 4$

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $c = 3$ and $a = 4$.

It is known that $a^2 = b^2 + c^2$.

$$a^2 = 8^2 + 6^2$$

$$= 64 + 36$$

$$= 100$$

$$16 = b^2 + 9$$

$$b^2 = 16 - 9$$

$$= 7$$

$\therefore$ The equation of the ellipse is $x^2/16 + y^2/7 = 1$

Question 18

$b = 3$, $c = 4$, centre at the origin; foci on the x axis.

Solution

Given:

$b = 3$, $c = 4$, centre at the origin and foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $b = 3$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$a^2 = 3^2 + 4^2$$

$$= 9 + 16$$

$$= 25$$

$$a = \sqrt{25}$$

$$= 5$$

∴ The equation of the ellipse is $x^2/5^2 + y^2/3^2$ or $x^2/25 + y^2/9 = 1$

Question 19

Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Solution

Given:

Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

The ellipse passes through points (3, 2) and (1, 6).

So, by putting the values $x = 3$ and $y = 2$, we get,

$$3^2/b^2 + 2^2/a^2 = 1$$

$$9/b^2 + 4/a^2 \dots (1)$$

And by putting the values $x = 1$ and $y = 6$, we get,

$$1^2/b^2 + 6^2/a^2 = 1$$

$$1/b^2 + 36/a^2 = 1 \dots (2)$$

On solving equations (1) and (2), we get

$$b^2 = 10 \text{ and } a^2 = 40.$$

∴ The equation of the ellipse is $x^2/10 + y^2/40 = 1$ or $4x^2 + y^2 = 40$

Question 20

Major axis on the x-axis and passes through the points (4,3) and (6,2).

Solution

Given:

Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Since the major axis is on the x-axis, the equation of the ellipse will be the form

$$x^2/a^2 + y^2/b^2 = 1 \dots (1) \text{ [Where 'a' is the semi-major axis.]}$$

The ellipse passes through points (4, 3) and (6, 2).

So by putting the values $x = 4$ and $y = 3$ in equation (1), we get,

$$16/a^2 + 9/b^2 = 1 \dots (2)$$

Putting $x = 6$ and $y = 2$ in equation (1), we get,

$$36/a^2 + 4/b^2 = 1 \dots (3)$$

From equation (2)

$$16/a^2 = 1 - 9/b^2$$

$$1/a^2 = (1/16 (1 - 9/b^2)) \dots (4)$$

Substituting the value of $1/a^2$ in equation (3), we get,

$$36/a^2 + 4/b^2 = 1$$

$$36(1/a^2) + 4/b^2 = 1$$

$$36[1/16 (1 - 9/b^2)] + 4/b^2 = 1$$

$$36/16 (1 - 9/b^2) + 4/b^2 = 1$$

$$9/4 (1 - 9/b^2) + 4/b^2 = 1$$

$$9/4 - 81/4b^2 + 4/b^2 = 1$$

$$-81/4b^2 + 4/b^2 = 1 - 9/4$$

$$(-81+16)/4b^2 = (4-9)/4$$

$$-65/4b^2 = -5/4$$

$$-5/4(13/b^2) = -5/4$$

$$13/b^2 = 1$$

$$1/b^2 = 1/13$$

$$b^2 = 13$$

Now substitute the value of b^2 in equation (4) we get,

$$1/a^2 = 1/16(1 - 9/b^2)$$

$$= 1/16(1 - 9/13)$$

$$= 1/16((13-9)/13)$$

$$= 1/16(4/13)$$

$$= 1/52$$

$$a^2 = 52$$

Equation of ellipse is $x^2/a^2 + y^2/b^2 = 1$

By substituting the values of a^2 and b^2 in the above equation, we get,

$$x^2/52 + y^2/13 = 1$$

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