

Chapter 11: Conic Sections Exercise 11.1

NCERT · Class 11 · Maths

Exercise 11.1

Question 1

Find the equation of the circle with centre (0,2) and radius 2

Solution

Ans: The equation of a circle with centre (h,k) and radius (r) is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (0, 2)$ and radius $(r) = 2$.

Therefore, the equation of the circle is

$$(x - 0)^2 + (y - 2)^2 = 2^2$$

$$x^2 + y^2 + 4 - 4y = 4$$

$$x^2 + y^2 - 4y = 0$$

Question 2

Find the equation of the circle with centre (-2,3) and radius 4

Solution

Ans: The equation of a circle with centre (h,k) and radius (r) is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h,k)=(-2,3)$ and radius $(r)=4$.

Therefore, the equation of the circle is

$$(x + 2)^2 + (y - 3)^2 = (4)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

Question 3

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\left(\frac{1}{12}\right)$

Solution

Ans: The equation of a circle with centre (h,k) and radius (r) is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = \left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $(r) = \left(\frac{1}{12}\right)$.

Therefore, the equation of the circle is

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} = \frac{1}{144}$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} - \frac{1}{144} = 0$$

$$144x^2 - 144x + 36 + 144y^2 - 72y + 9 - 1 = 0 \text{ (Solve by taking LCM)}$$

$$144x^2 - 144x + 144y^2 - 72y + 44 = 0$$

$$36x^2 - 36x + 36y^2 - 18y + 11 = 0$$

$$36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

Question 4

Find the equation of the circle with centre (1,1) and radius $\sqrt{2}$

Solution

Ans: The equation of a circle with centre (h,k) and radius (r) is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre (h,k)=(1,1) and radius $(r) = \sqrt{2}$.

Therefore, the equation of the circle is

$$(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$

$$x^2 + y^2 - 2x - 2y = 0$$

Question 5

Find the equation of the circle with centre (-a,-b) and radius $\sqrt{a^2 - b^2}$

Solution

Ans: The equation of a circle with centre (h,k) and radius (r) is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre (h,k)=(-a,-b) and radius $(r) = \sqrt{a^2 - b^2}$.

Therefore, the equation of the circle is

$$(x + a)^2 + (y + b)^2 = (\sqrt{a^2 - b^2})^2$$

$$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$$

$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

Question 6

Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

Solution

Ans: The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$

$$(x + 5)^2 + (y - 3)^2 = 36$$

$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = (6)^2$, which is the form of $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5, k = 3$, and $r = 6$.

Thus, the centre of the given circle is $(-5, 3)$, while its radius is 6.

Question 7

Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Solution

Ans: The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$

$$\Rightarrow x^2 + y^2 - 4x - 8y - 45 = 0$$

$$\Rightarrow (x^2 - 4x) + (y^2 - 8y) = 45$$

$$\Rightarrow \{x^2 - 2(x)(2) + (2)^2\} + \{y^2 - 2(y)(4) + (4)^2\} - 4 - 16 = 45$$

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = 65$$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = (\sqrt{65})^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2, k = 4$, and $r = \sqrt{65}$

Thus, the centre of the given circle is $(2, 4)$, while its radius is $\sqrt{65}$.

Question 8

Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

Solution

Ans: The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$

$$\Rightarrow x^2 + y^2 - 8x + 10y - 12 = 0$$

$$\Rightarrow (x^2 - 8x) + (y^2 + 10y) = 12$$

$$\Rightarrow \{x^2 - 2(x)(4) + (4)^2\} + \{y^2 + 2(y)(5) + (5)^2\} - 16 - 25 = 12$$

$$\Rightarrow (x - 4)^2 + (y + 5)^2 = 53$$

$$\Rightarrow (x - 4)^2 + \{y - (-5)\}^2 = (\sqrt{53})^2, \text{ which is of the form } (x - h)^2 + (y - k)^2 = r^2, \text{ where } h = 4, k = -5, \text{ and } r = \sqrt{53}$$

Thus, the centre of the given circle is $(4, -5)$, while its radius is $\sqrt{53}$.

Question 9

Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Solution

Ans: The equation of the given circle is $2x^2 + 2y^2 - x = 0$

$$\Rightarrow 2x^2 + 2y^2 - x = 0$$

$$\Rightarrow (2x^2 - x) + 2y^2 = 0$$

$$\Rightarrow 2 \left[\left(x^2 - \frac{x}{2} \right) + y^2 \right] = 0$$

$$\Rightarrow \left\{ x^2 - 2.x \left(\frac{1}{4} \right) + \left(\frac{1}{4} \right)^2 \right\} + y^2 - \left(\frac{1}{4} \right)^2 = 0$$

$$\Rightarrow \left(x - \frac{1}{4} \right)^2 + (y - 0)^2 = \left(\frac{1}{4} \right)^2, \text{ which is of the form } (x - h)^2 + (y - k)^2 = r^2, \text{ where } h = \frac{1}{4}, k = 0, \text{ and } r = \frac{1}{4}$$

Thus, the centre of the given circle is $\left(\frac{1}{4}, 0 \right)$, while its radius is $\frac{1}{4}$.

Question 10

Find the equation of the circle passing through the points $(4, 1)$ and $(6, 5)$ and whose centre is on the line $4x + y = 16$

Solution

Ans: Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through points $(4, 1)$ and $(6, 5)$,

$$(4 - h)^2 + (1 - k)^2 = r^2 \dots\dots\dots(i)$$

$$(6 - h)^2 + (5 - k)^2 = r^2 \dots\dots\dots(ii)$$

Since the centre (h, k) of the circle lies on line $4x + y = 16$,

$$4h + k = 16 \dots\dots\dots(iii)$$

From equations (i) and (ii), we get

$$\Rightarrow (4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$$

$$\Rightarrow 16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$$

$$\Rightarrow 16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$$

$$\Rightarrow 4h + 8k = 44$$

$$\Rightarrow h + 2k = 11 \dots\dots\dots(iv)$$

On solving equations (iii) and (iv), we obtain $h = 3$ and $k = 4$

On substituting the values of h and k in equation (i), we obtain

$$(4 - 3)^2 + (1 - 4)^2 = r^2$$

$$\Rightarrow (1)^2 + (-3)^2 = r^2$$

$$\Rightarrow 1 + 9 = r^2$$

$$\Rightarrow r^2 = 10$$

$$\Rightarrow r = \sqrt{10}$$

Thus, the equation of the required circle is

$$(x - 3)^2 + (y - 4)^2 = (\sqrt{10})^2$$

$$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$$

$$x^2 + y^2 - 6x - 8y + 15 = 0$$

Question 11

Find the equation of the circle passing through the points (2,3) and (-1,1) and whose centre is on the line $x - 3y - 11 = 0$

Solution

Ans: Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through points (2, 3) and (-1, 1),

$$(2 - h)^2 + (3 - k)^2 = r^2 \dots\dots\dots(i)$$

$$(-1 - h)^2 + (1 - k)^2 = r^2 \dots\dots\dots(ii)$$

Since the centre (h,k) of the circle lies on line $x - 3y - 11 = 0$

$$h - 3k = 11 \dots\dots\dots(iii)$$

From equations (i) and (ii), we get

$$\Rightarrow (2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$$

$$\Rightarrow 4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$$

$$\Rightarrow 4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$$

$$\Rightarrow 6h + 4k = 11 \dots\dots\dots(iv)$$

On solving equations (iii) and (iv), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$

On substituting the values of h and k in equation (i), we obtain

$$\Rightarrow \left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{4 - 7}{2}\right)^2 + \left(\frac{6 + 5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{-3}{2}\right)^2 + \left(\frac{11}{2}\right)^2 = r^2$$

$$\Rightarrow \frac{9}{4} + \frac{121}{4} = r^2$$

$$\Rightarrow \frac{130}{4} = r^2$$

Thus, the equation of the required circle is

$$\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2 + \left(\frac{2y+5}{2}\right)^2 = \frac{130}{4}$$

$$4x^2 - 28x + 49 + 4y^2 + 20y + 25 = 130$$

$$4x^2 + 4y^2 - 28x + 20y - 56 = 0$$

$$4(x^2 + y^2 - 7x + 5y - 14) = 0$$

$$x^2 + y^2 - 7x + 5y - 14 = 0$$

Question 12

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2,3).

Solution

Ans: Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the radius of the circle is 5 and its centre lies on the x-axis, $k=0$ and $r=5$.

Now, the equation of the circle passes through point (2, 3).

$$\therefore (2 - h)^2 + 3^2 = 25$$

$$\Rightarrow (2 - h)^2 = 25 - 9$$

$$\Rightarrow (2 - h)^2 = 16$$

$$\Rightarrow (2 - h) = \pm\sqrt{16}$$

$$= \pm 4$$

If $2 - h = 4$, then $h = -2$

If $2 - h = -4$, then $h = 6$

When $h = -2$, the equation of the circle becomes

$$(x + 2)^2 + y^2 = 25$$

$$x^2 + 4x + 4 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

When $h = 6$, the equation of the circle becomes

$$(x - 6)^2 + y^2 = 25$$

$$x^2 - 12x + 36 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

So, the equation of the circle can be $x^2 + y^2 + 4x - 21 = 0$ or $x^2 + y^2 - 12x + 11 = 0$

Question 13

Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axes.

Solution

Ans: Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through (0,0).

$$(0 - h)^2 + (0 - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 = r^2$$

The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points (a, 0) and (0, b). Therefore,

$$(a - h)^2 + (0 - k)^2 = h^2 + k^2 \dots\dots(1)$$

$$(0 - h)^2 + (b - k)^2 = h^2 + k^2 \dots\dots(2)$$

From equation (1), we'll get

$$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$$

$$\Rightarrow a^2 - 2ah = 0$$

$$\Rightarrow a(a - 2h) = 0$$

$$\Rightarrow a = 0 \text{ or } (a - 2h) = 0$$

$$\text{However, } a \neq 0; \text{ hence, } (a - 2h) = 0 \Rightarrow h = \frac{a}{2}$$

From equation (2), we'll get

$$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$$

$$\Rightarrow b^2 - 2bk = 0$$

$$\Rightarrow b(b - 2k) = 0$$

$$\Rightarrow b = 0 \text{ or } (b - 2k) = 0$$

$$\text{However, } b \neq 0; \text{ hence, } (b - 2k) = 0 \Rightarrow k = \frac{b}{2}$$

Thus, the equation of the required circle is

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$$

$$\left(\frac{2x - a}{2}\right)^2 + \left(\frac{2y - b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

$$\Rightarrow 4x^2 - 4ax + a^2 + 4y^2 - 4by + b^2 = a^2 + b^2$$

$$\Rightarrow 4x^2 + 4y^2 - 4ax - 4by = 0$$

$$\Rightarrow x^2 + y^2 - ax - by = 0$$

Question 14

Find the equation of a circle with centre (2,2) and passes through the point (4,5).

Solution

Ans: The centre of the circle is given as (h,k)=(2,2)

Since the circle passes through point (4,5), the radius (r) of the circle is the distance between the points (2,2) and (4,5).

$$\begin{aligned}\therefore r &= \sqrt{(2-4)^2 + (2-5)^2} \\ &= \sqrt{(-2)^2 + (-3)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13}\end{aligned}$$

Thus, the equation of the circle is

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-2)^2 + (y-2)^2 &= (\sqrt{13})^2 \\ x^2 - 4x + 4 + y^2 - 4y + 4 &= 13 \\ x^2 + y^2 - 4x - 4y - 5 &= 0\end{aligned}$$

Question 15

Does the point (-2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Solution

Ans: The equation of the given circle is $x^2 + y^2 = 25$.

$$x^2 + y^2 = 25$$

$\Rightarrow (x-0)^2 + (y-0)^2 = 5^2$, which is of the form of $(x-h)^2 + (y-k)^2 = r^2$, where $h=0, k=0$ and $r=5$

$\therefore$ Centre = (0,0) and radius = 5

Distance between point (-2.5, 3.5) and centre (0,0)

$$\begin{aligned}&= \sqrt{(-2.5-0)^2 + (3.5-0)^2} \\ &= \sqrt{6.25 + 12.25} \\ &= \sqrt{18.25} \\ &= 4.3 \text{ (approx.)} < 5\end{aligned}$$

Since the distance between point (-2.5, 3.5) and centre (0,0) of the circle is less than the radius of the circle, point (-2.5, 3.5) lies inside the circle.

