

Chapter 10: Straight Lines Miscellaneous Exercise

NCERT · Class 11 · Maths

Questions

Question 1

Find the values of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is

- (a) Parallel to the x-axis
- (b) Parallel to the y-axis
- (c) Passing through the origin

Solution

It is given that

$$(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0 \dots (1)$$

(a) Here, if the line is parallel to the x-axis,

The slope of the line = Slope of the x-axis

It can be written as

$$(4 - k^2)y = (k - 3)x + k^2 - 7k + 6 = 0$$

We get,

$$y = \frac{(k - 3)}{(4 - k^2)}x + \frac{k^2 - 7k + 6}{(4 - k^2)}$$

Which is of the form $y = mx + c$

Here the slope of the given line

$$= \frac{(k - 3)}{(4 - k^2)}$$

Consider the slope of x-axis = 0

$$\frac{(k - 3)}{(4 - k^2)} = 0$$

By further calculation,

$$k - 3 = 0$$

$$k = 3$$

Hence, if the given line is parallel to the x-axis, then the value of k is 3.

(b) Here, if the line is parallel to the y-axis, it is vertical, and the slope will be undefined.

So, the slope of the given line

$$= \frac{(k-3)}{(4-k^2)}$$

Here,

$$\frac{(k-3)}{(4-k^2)} \text{ is undefined at } k^2 = 4$$

$$k^2 = 4$$

$$k = \pm 2$$

Hence, if the given line is parallel to the y-axis, then the value of k is ± 2 .

(c) Here, if the line is passing through (0, 0), which is the origin satisfies the given equation of the line.

$$(k-3)(0) - (4-k^2)(0) + k^2 - 7k + 6 = 0$$

By further calculation,

$$k^2 - 7k + 6 = 0$$

Separating the terms,

$$k^2 - 6k - k + 6 = 0$$

We get

$$(k-6)(k-1) = 0$$

$$k = 1 \text{ or } 6$$

Hence, if the given line is passing through the origin, then the value of k is either 1 or 6.

Question 2

Find the values of θ and p , if the equation $x \cos \theta + y \sin \theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Solution

It is given that

$$\sqrt{3}x + y + 2 = 0$$

It can be reduced as

$$\sqrt{3}x + y + 2 = 0$$

$$-\sqrt{3}x - y = 2$$

By dividing both sides by $\sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$, we get

$$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y = \frac{2}{2}$$

It can be written as

$$\left(-\frac{\sqrt{3}}{2}\right)x + \left(-\frac{1}{2}\right)y = 1 \quad \dots(1)$$

By comparing equation (1) to $x \cos \theta + y \sin \theta = p$, we get

$$\cos \theta = -\frac{\sqrt{3}}{2}, \quad \sin \theta = -\frac{1}{2}, \quad \text{and } p = 1$$

Here the values of $\sin \theta$ and $\cos \theta$ are negative

$$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Hence, the respective values of θ and p are $7\pi/6$ and 1.

Question 3

Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6, respectively.

Solution

Consider the intercepts cut by the given lines on the a and b axes.

$$a + b = 1 \quad \dots\dots (1)$$

$$ab = -6 \quad \dots\dots (2)$$

By solving both equations, we get

$$a = 3 \text{ and } b = -2 \text{ or } a = -2 \text{ and } b = 3$$

We know that the equation of the line whose intercepts on the a and b axes is

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ or } bx + ay - ab = 0$$

Case I – $a = 3$ and $b = -2$

So, the equation of the line is $-2x + 3y + 6 = 0$, i.e. $2x - 3y = 6$.

Case II – $a = -2$ and $b = 3$

So, the equation of the line is $3x - 2y + 6 = 0$, i.e. $-3x + 2y = 6$

Hence, the required equation of the lines are $2x - 3y = 6$ and $-3x + 2y = 6$.

Question 4

What are the points on the y-axis whose distance from the line $x/3 + y/4 = 1$ is 4 units?

Solution

Consider $(0, b)$ as the point on the y-axis whose distance from line $x/3 + y/4 = 1$ is 4 units.

It can be written as $4x + 3y - 12 = 0$ (1)

By comparing equation (1) to the general equation of line $Ax + By + C = 0$, we get

$$A = 4, B = 3 \text{ and } C = -12$$

We know that the perpendicular distance (d) of a line $Ax + By + C = 0$ from (x_1, y_1) is written as

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

If $(0, b)$ is the point on the y-axis whose distance from line $x/3 + y/4 = 1$ is 4 units, then

$$4 = \frac{|4(0) + 3(b) - 12|}{\sqrt{4^2 + 3^2}}$$

By further calculation

$$4 = \frac{|3b - 12|}{5}$$

By cross multiplication

$$20 = |3b - 12|$$

We get

$$20 = \pm (3b - 12)$$

$$\text{Here, } 20 = (3b - 12) \text{ or } 20 = -(3b - 12)$$

It can be written as

$$3b = 20 + 12 \text{ or } 3b = -20 + 12$$

So, we get

$$b = 32/3 \text{ or } b = -8/3$$

Hence, the required points are $(0, 32/3)$ and $(0, -8/3)$.

Question 5

Find the perpendicular distance from the origin to the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$.

Solution

Here the equation of the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ is written as

$$y - \sin \theta = \frac{\sin \phi - \sin \theta}{\cos \phi - \cos \theta} (x - \cos \theta)$$

By cross multiplication

$$y(\cos \phi - \cos \theta) - \sin \theta(\cos \phi - \cos \theta) = x(\sin \phi - \sin \theta) - \cos \theta(\sin \phi - \sin \theta)$$

By multiplying the terms we get

$$x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \cos \theta \sin \phi - \cos \theta \sin \theta - \sin \theta \cos \phi + \sin \theta \cos \theta = 0$$

On further simplification

$$x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \sin(\phi - \theta) = 0$$

So we get

$$Ax + By + C = 0, \text{ where } A = \sin \theta - \sin \phi, B = \cos \phi - \cos \theta, \text{ and } C = \sin(\phi - \theta)$$

We know that the perpendicular distance (d) of a line $Ax + By + C = 0$ from (x_1, y_1) is written as

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

So the perpendicular distance (d) of the given line from $(x_1, y_1) = (0, 0)$ is

$$d = \frac{|(\sin \theta - \sin \phi)(0) + (\cos \phi - \cos \theta)(0) + \sin(\phi - \theta)|}{\sqrt{(\sin \theta - \sin \phi)^2 + (\cos \phi - \cos \theta)^2}}$$

By expanding using formula

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{\sin^2 \theta + \sin^2 \phi - 2 \sin \theta \sin \phi + \cos^2 \phi + \cos^2 \theta - 2 \cos \phi \cos \theta}}$$

Grouping of terms

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{(\sin^2 \theta + \cos^2 \theta) + (\sin^2 \phi + \cos^2 \phi) - 2(\sin \theta \sin \phi + \cos \theta \cos \phi)}}$$

By further simplification

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{1 + 1 - 2(\cos(\phi - \theta))}}$$

Taking out 2 as common

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{2(1 - \cos(\phi - \theta))}}$$

Using the formula

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{2\left(2\sin^2\left(\frac{\phi - \theta}{2}\right)\right)}}$$

We get

$$= \frac{|\sin(\phi - \theta)|}{\left|2\sin\left(\frac{\phi - \theta}{2}\right)\right|}$$

Question 6

Find the equation of the line parallel to the y-axis drawn through the point of intersection of the lines $x - 7y + 5 = 0$ and $3x + y = 0$.

Solution

Here, the equation of any line parallel to the y-axis is of the form,

$$x = a \dots\dots (1)$$

Two given lines are

$$x - 7y + 5 = 0 \dots\dots (2)$$

$$3x + y = 0 \dots\dots (3)$$

By solving equations (2) and (3), we get

$$x = -5/22 \text{ and } y = 15/22$$

$(-5/22, 15/22)$ is the point of intersection of lines (2) and (3)

If the line $x = a$ passes through point $(-5/22, 15/22)$, we get $a = -5/22$

Hence, the required equation of the line is $x = -5/22$.

Question 7

Find the equation of a line drawn perpendicular to the line $x/4 + y/6 = 1$ through the point where it meets the y-axis.

Solution

It is given that

$$x/4 + y/6 = 1$$

We can write it as

$$3x + 2y - 12 = 0$$

So, we get

$$y = -3/2 x + 6, \text{ which is of the form } y = mx + c$$

Here, the slope of the given line = $-3/2$

So, the slope of line perpendicular to the given line = $-1/(-3/2) = 2/3$

Consider the given line intersects the y-axis at $(0, y)$

By substituting x as zero in the equation of the given line,

$$y/6 = 1$$

$$y = 6$$

Hence, the given line intersects the y-axis at $(0, 6)$

We know that the equation of the line that has a slope of $2/3$ and passes through the point $(0, 6)$ is

$$(y - 6) = 2/3 (x - 0)$$

By further calculation,

$$3y - 18 = 2x$$

So, we get

$$2x - 3y + 18 = 0$$

Hence, the required equation of the line is $2x - 3y + 18 = 0$.

Question 8

Find the area of the triangle formed by the lines $y - x = 0$, $x + y = 0$ and $x - k = 0$.

Solution

It is given that

$$y - x = 0 \dots\dots (1)$$

$$x + y = 0 \dots\dots (2)$$

$$x - k = 0 \dots\dots (3)$$

Here, the point of intersection of

Lines (1) and (2) is

$$x = 0 \text{ and } y = 0$$

Lines (2) and (3) is

$$x = k \text{ and } y = -k$$

Lines (3) and (1) is

$$x = k \text{ and } y = k$$

So, the vertices of the triangle formed by the three given lines are $(0, 0)$, $(k, -k)$ and (k, k)

Here, the area of triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is

$$\frac{1}{2} |x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$$

So, the area of the triangle formed by the three given lines

$$= \frac{1}{2} |0 (-k - k) + k (k - 0) + k (0 + k)| \text{ square units}$$

By further calculation,

$$= \frac{1}{2} |k^2 + k^2| \text{ square units}$$

So, we get

$$= \frac{1}{2} |2k^2|$$

$$= k^2$$

Hence, the area of the triangle formed by the three given lines is k^2 .

Question 9

Find the value of p so that the three lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

Solution

It is given that

$$3x + y - 2 = 0 \dots\dots (1)$$

$$px + 2y - 3 = 0 \dots\dots (2)$$

$$2x - y - 3 = 0 \dots\dots (3)$$

By solving equations (1) and (3), we get

$$x = 1 \text{ and } y = -1$$

Here, the three lines intersect at one point, and the point of intersection of lines (1) and (3) will also satisfy line (2)

$$p(1) + 2(-1) - 3 = 0$$

By further calculation,

$$p - 2 - 3 = 0$$

So, we get

$$p = 5$$

Hence, the required value of p is 5.

Question 10

If three lines whose equations are $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ are concurrent, then show that $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$.

Solution

It is given that

$$y = m_1x + c_1 \dots\dots (1)$$

$$y = m_2x + c_2 \dots\dots (2)$$

$$y = m_3x + c_3 \dots\dots (3)$$

By subtracting equation (1) from (2), we get

$$0 = (m_2 - m_1)x + (c_2 - c_1)$$

$$(m_1 - m_2)x = c_2 - c_1$$

So, we get

$$x = \frac{c_2 - c_1}{m_1 - m_2}$$

By substituting this value in equation (1) we get

$$y = m_1 \left(\frac{c_2 - c_1}{m_1 - m_2} \right) + c_1$$

By multiplying the terms

$$y = \frac{m_1c_2 - m_1c_1}{m_1 - m_2} + c_1$$

Taking LCM

$$y = \frac{m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1}{m_1 - m_2}$$

On further simplification

$$y = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

Here

$$\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2} \right) \text{ is the point of intersection of lines (1) and (2)}$$

Lines (1), (2) and (3) are concurrent. So the point of intersection of lines (1) and (2) will satisfy equation (3)

$$\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} = m_3 \left(\frac{c_2 - c_1}{m_1 - m_2} \right) + c_3$$

By multiplying the terms and taking LCM

$$\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} = \frac{m_3 c_2 - m_3 c_1 + c_3 m_1 - c_3 m_2}{m_1 - m_2}$$

By cross multiplication

$$m_1 c_2 - m_2 c_1 - m_3 c_2 + m_3 c_1 - c_3 m_1 + c_3 m_2 = 0$$

Taking out the common terms

$$m_1 (c_2 - c_3) + m_2 (c_3 - c_1) + m_3 (c_1 - c_2) = 0$$

$$\text{Therefore, } m_1 (c_2 - c_3) + m_2 (c_3 - c_1) + m_3 (c_1 - c_2) = 0.$$

Question 11

Find the equation of the lines through the point (3, 2), which makes an angle of 45° with the line $x - 2y = 3$.

Solution

Consider m_1 as the slope of the required line

It can be written as

$$y = \frac{1}{2}x - \frac{3}{2} \text{ which is of the form } y = mx + c$$

So, the slope of the given line $m_2 = 1/2$

We know that the angle between the required line and line $x - 2y = 3$ is 45°

If θ is the acute angle between lines l_1 and l_2 with slopes m_1 and m_2 ,

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

We get

$$\tan 45^\circ = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Substituting the values

$$1 = \left| \frac{\frac{1}{2} - m_1}{1 + \frac{m_1}{2}} \right|$$

By taking LCM

$$1 = \left| \frac{\left(\frac{1 - 2m_1}{2} \right)}{\frac{2 + m_1}{2}} \right|$$

On further calculation

$$1 = \left| \frac{1 - 2m_1}{2 + m_1} \right|$$

We get

$$1 = \pm \left(\frac{1 - 2m_1}{2 + m_1} \right)$$

Here

$$1 = \frac{1 - 2m_1}{2 + m_1} \text{ or } 1 = - \left(\frac{1 - 2m_1}{2 + m_1} \right)$$

It can be written as

$$2 + m_1 = 1 - 2m_1 \text{ or } 2 + m_1 = -1 + 2m_1$$

$$m_1 = -1/3 \text{ or } m_1 = 3$$

Case I – $m_1 = 3$

Here, the equation of the line passing through (3, 2) and having a slope of 3 is

$$y - 2 = 3(x - 3)$$

By further calculation,

$$y - 2 = 3x - 9$$

So, we get

$$3x - y = 7$$

$$\text{Case II} - m_1 = -1/3$$

Here, the equation of the line passing through (3, 2) and having a slope of $-1/3$ is

$$y - 2 = -1/3 (x - 3)$$

By further calculation,

$$3y - 6 = -x + 3$$

So, we get

$$x + 3y = 9$$

Hence, the equations of the lines are $3x - y = 7$, and $x + 3y = 9$.

Question 12

Find the equation of the line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

Solution

Consider the equation of the line having equal intercepts on the axes as

$$x/a + y/a = 1$$

It can be written as

$$x + y = a \dots (1)$$

By solving equations $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$, we get

$$x = 1/13 \text{ and } y = 5/13$$

$(1/13, 5/13)$ is the point of intersection of two given lines.

We know that equation (1) passes through the point $(1/13, 5/13)$

$$1/13 + 5/13 = a$$

$$a = 6/13$$

So, equation (1) passes through $(1/13, 5/13)$.

$$1/13 + 5/13 = a$$

We get

$$a = 6/13$$

Here, equation (1) becomes

$$x + y = 6/13$$

$$13x + 13y = 6$$

Hence, the required equation of the line is $13x + 13y = 6$.

Question 13

Show that the equation of the line passing through the origin and making an angle θ with the line $y =$

$$mx + c \text{ is } \frac{y}{x} = \frac{m \pm \tan \theta}{1 \mp m \tan \theta}.$$

Solution

Consider $y = m_1x$ as the equation of the line passing through the origin

It is given that the line makes an angle θ with line $y = mx + c$, then angle θ is written as

$$\tan \theta = \left| \frac{m_1 - m}{1 + m_1 m} \right|$$

By substituting the values

$$\tan \theta = \left| \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right|$$

We get

$$\tan \theta = \pm \left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)$$

Here

$$\tan \theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \quad \text{or} \quad \tan \theta = - \left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)$$

Case I –

$$\tan \theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}$$

We can write it as

$$\tan \theta + \frac{y}{x} m \tan \theta = \frac{y}{x} - m$$

By further simplification

$$m + \tan \theta = \frac{y}{x} (1 - m \tan \theta)$$

So we get

$$\frac{y}{x} = \frac{m + \tan \theta}{1 - m \tan \theta}$$

Case II –

$$\tan \theta = - \left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)$$

We can write it as

$$\tan \theta + \frac{y}{x} m \tan \theta = -\frac{y}{x} + m$$

By further simplification

$$\frac{y}{x}(1 + m \tan \theta) = m - \tan \theta$$

So we get

$$\frac{y}{x} = \frac{m - \tan \theta}{1 + m \tan \theta}$$

Hence, the required line is given by

$$\frac{y}{x} = \frac{m \pm \tan \theta}{1 \mp m \tan \theta}$$

Question 14

In what ratio the line joining $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$?

Solution

We know that the equation of the line joining the points $(-1, 1)$ and $(5, 7)$ is given by

$$y - 1 = \frac{7 - 1}{5 - (-1)}(x + 1)$$

By further calculation

$$y - 1 = \frac{6}{6}(x + 1)$$

So we get

$$x - y + 2 = 0 \dots\dots (1)$$

So the equation of the given line is

$$x + y - 4 = 0 \dots\dots (2)$$

Here the point of intersection of lines (1) and (2) is given by

$$x = 1 \text{ and } y = 3$$

Consider (1, 3) divide the line segment joining (-1, 1) and (5, 7) in the ratio 1: k.

Using the section formula

$$(1, 3) = \left(\frac{k(-1) + 1(5)}{1 + k}, \frac{k(1) + 1(7)}{1 + k} \right)$$

By further calculation

$$(1, 3) = \left(\frac{-k + 5}{1 + k}, \frac{k + 7}{1 + k} \right)$$

So we get

$$\frac{-k + 5}{1 + k} = 1, \frac{k + 7}{1 + k} = 3$$

We can write it as

$$\frac{-k + 5}{1 + k} = 1$$

By cross multiplication

$$-k + 5 = 1 + k$$

We get

$$2k = 4$$

$$k = 2$$

Hence, the line joining the points (-1, 1) and (5, 7) is divided by the line $x + y = 4$ in the ratio 1:2.

Question 15

Find the distance of the line $4x + 7y + 5 = 0$ from the point (1, 2) along the line $2x - y = 0$.

Solution

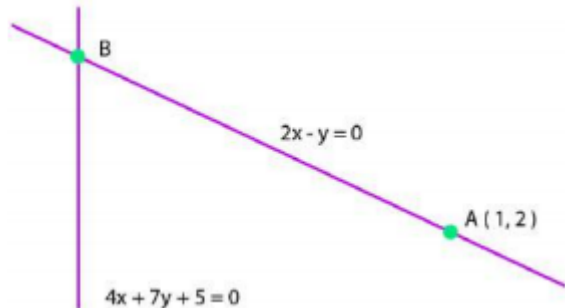
It is given that

$$2x - y = 0 \dots\dots (1)$$

$$4x + 7y + 5 = 0 \dots\dots (2)$$

Here A (1, 2) is a point on line (1)

Consider B as the point of intersection of lines (1) and (2)



By solving equations (1) and (2), we get $x = -5/18$ and $y = -5/9$

So, the coordinates of point B is $(-5/18, -5/9)$

From the distance formula, the distance between A and B

$$AB = \sqrt{\left(1 + \frac{5}{18}\right)^2 + \left(2 + \frac{5}{9}\right)^2} \text{ units}$$

By taking LCM

$$= \sqrt{\left(\frac{23}{18}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

It can be written as

$$= \sqrt{\left(\frac{23}{2 \times 9}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

So we get

$$= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{2}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

By taking the common terms out

$$= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{4} + 1\right)} \text{ units}$$

We get

$$= \frac{23}{9} \sqrt{\frac{5}{4}} \text{ units}$$

$$= \frac{23}{9} \times \frac{\sqrt{5}}{2} \text{ units}$$

So we get

$$= \frac{23\sqrt{5}}{18} \text{ units}$$

Hence, the required distance is

$$\frac{23\sqrt{5}}{18} \text{ units}.$$

Question 16

Find the direction in which a straight line must be drawn through the point $(-1, 2)$ so that its point of intersection with the line $x + y = 4$ may be at a distance of 3 units from this point.

Solution

Consider $y = mx + c$ as the line passing through the point $(-1, 2)$

So, we get

$$2 = m(-1) + c$$

By further calculation,

$$2 = -m + c$$

$$c = m + 2$$

Substituting the value of c ,

$$y = mx + m + 2 \dots\dots (1)$$

So, the given line is

$$x + y = 4 \dots\dots (2)$$

By solving both equations, we get

$$x = \frac{2-m}{m+1} \text{ and } y = \frac{5m+2}{m+1}$$

$\left(\frac{2-m}{m+1}, \frac{5m+2}{m+1}\right)$ is the point of intersection of lines (1) and (2)

Here the point is at a distance of 3 units from $(-1, 2)$

From distance formula

$$\sqrt{\left(\frac{2-m}{m+1} + 1\right)^2 + \left(\frac{5m+2}{m+1} - 2\right)^2} = 3$$

Squaring on both sides

$$\left(\frac{2-m+m+1}{m+1}\right)^2 + \left(\frac{5m+2-2m-2}{m+1}\right)^2 = 3^2$$

By further calculation

$$\frac{9}{(m+1)^2} + \frac{9m^2}{(m+1)^2} = 9$$

Dividing the equation by 9

$$\frac{1+m^2}{(m+1)^2} = 1$$

By cross multiplication

$$1 + m^2 = m^2 + 1 + 2m$$

So, we get

$$2m = 0$$

$$m = 0$$

Hence, the slope of the required line must be zero, i.e. the line must be parallel to the x-axis.

Question 17

The hypotenuse of a right-angled triangle has its ends at points $(1, 3)$ and $(-4, 1)$. Find the equation of the legs (perpendicular sides) of the triangle.

Solution

Consider ABC as the right angles triangle where $\angle C = 90^\circ$

Here, infinitely such lines are present.

m is the slope of AC

So, the slope of BC = $-1/m$

Equation of AC,

$$y - 3 = m(x - 1)$$

By cross multiplication,

$$x - 1 = 1/m(y - 3)$$

Equation of BC,

$$y - 1 = -1/m(x + 4)$$

By cross multiplication,

$$x + 4 = -m(y - 1)$$

By considering the values of m , we get

If $m = 0$,

So, we get

$$y - 3 = 0, x + 4 = 0$$

If $m = \infty$,

So, we get

$$x - 1 = 0, y - 1 = 0 \text{ we get } x = 1, y = 1$$

Question 18

Find the image of the point (3, 8) with respect to the line $x + 3y = 7$, assuming the line to be a plane mirror.

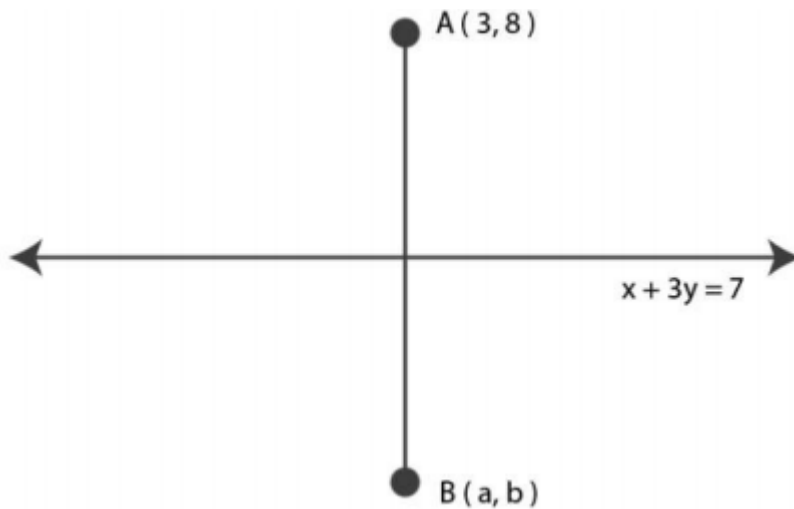
Solution

It is given that

$$x + 3y = 7 \dots (1)$$

Consider B (a, b) as the image of point A (3, 8)

So, line (1) is the perpendicular bisector of AB.



Here

$$\text{Slope of AB} = \frac{b-8}{a-3}$$

$$\text{slope of line (1)} = -\frac{1}{3}$$

Line (1) is perpendicular to AB

$$\left(\frac{b-8}{a-3}\right) \times \left(-\frac{1}{3}\right) = -1$$

By further calculation

$$\frac{b-8}{3a-9} = 1$$

By cross multiplication

$$b-8 = 3a-9$$

$$3a-b = 1 \dots\dots (2)$$

We know that

$$\text{Mid-point of AB} = \left(\frac{a+3}{2}, \frac{b+8}{2}\right)$$

So the mid-point of line segment AB will satisfy line (1)

From equation (1)

$$\left(\frac{a+3}{2}\right) + 3\left(\frac{b+8}{2}\right) = 7$$

By further calculation

$$a + 3 + 3b + 24 = 14$$

On further simplification,

$$a + 3b = -13 \dots (3)$$

By solving equations (2) and (3), we get

$$a = -1 \text{ and } b = -4$$

Hence, the image of the given point with respect to the given line is (-1, -4).

Question 19

If the lines $y = 3x + 1$, and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of m .

Solution

It is given that

$$y = 3x + 1 \dots (1)$$

$$2y = x + 3 \dots (2)$$

$$y = mx + 4 \dots (3)$$

Here, the slopes of

$$\text{Line (1), } m_1 = 3$$

$$\text{Line (2), } m_2 = \frac{1}{2}$$

$$\text{Line (3), } m_3 = m$$

We know that lines (1) and (2) are equally inclined to the line (3), which means that the angle between lines (1) and (3) equals the angle between lines (2) and (3).

$$\left| \frac{m_1 - m_3}{1 + m_1 m_3} \right| = \left| \frac{m_2 - m_3}{1 + m_2 m_3} \right|$$

Substituting the values we get

$$\left| \frac{3 - m}{1 + 3m} \right| = \left| \frac{\frac{1}{2} - m}{1 + \frac{1}{2}m} \right|$$

By taking LCM

$$\left| \frac{3 - m}{1 + 3m} \right| = \left| \frac{1 - 2m}{m + 2} \right|$$

It can be written as

$$\frac{3 - m}{1 + 3m} = \pm \left(\frac{1 - 2m}{m + 2} \right)$$

Here

$$\frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2} \text{ or } \frac{3 - m}{1 + 3m} = - \left(\frac{1 - 2m}{m + 2} \right)$$

If

$$\frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2}$$

By cross multiplication

$$(3 - m)(m + 2) = (1 - 2m)(1 + 3m)$$

On further calculation,

$$-m^2 + m + 6 = 1 + m - 6m^2$$

So we get

$$5m^2 + 5 = 0$$

Dividing the equation by 5,

$$m^2 + 1 = 0$$

$m = \sqrt{-1}$, which is not real.

Therefore, this case is not possible.

If

$$\frac{3-m}{1+3m} = -\left(\frac{1-2m}{m+2}\right)$$

By cross multiplication

$$(3-m)(m+2) = -(1-2m)(1+3m)$$

On further calculation

$$-m^2 + m + 6 = -(1 + m - 6m^2)$$

So we get

$$7m^2 - 2m - 7 = 0$$

Here we get

$$m = \frac{2 \pm \sqrt{4 - 4(7)(-7)}}{2(7)}$$

By further simplification

$$m = \frac{2 \pm 2\sqrt{1+49}}{14}$$

We can write it as

$$m = \frac{1 \pm 5\sqrt{2}}{7}$$

Hence, the required value of m is

$$\frac{1 \pm 5\sqrt{2}}{7}$$

Question 20

If the sum of the perpendicular distances of a variable point P (x, y) from the lines $x + y - 5 = 0$, and $3x - 2y + 7 = 0$, is always 10, show that P must move on a line.

Solution

It is given that

$$x + y - 5 = 0 \dots (1)$$

$$3x - 2y + 7 = 0 \dots (2)$$

Here the perpendicular distances of P (x, y) from lines (1) and (2) are written as

$$d_1 = \frac{|x + y - 5|}{\sqrt{(1)^2 + (1)^2}} \text{ and } d_2 = \frac{|3x - 2y + 7|}{\sqrt{(3)^2 + (-2)^2}}$$

So we get

$$d_1 = \frac{|x + y - 5|}{\sqrt{2}} \text{ and } d_2 = \frac{|3x - 2y + 7|}{\sqrt{13}}$$

We know that $d_1 + d_2 = 10$

Substituting the values

$$\frac{|x + y - 5|}{\sqrt{2}} + \frac{|3x - 2y + 7|}{\sqrt{13}} = 10$$

By further calculation

$$\sqrt{13}|x + y - 5| + \sqrt{2}|3x - 2y + 7| - 10\sqrt{26} = 0$$

It can be written as

$$\sqrt{13}(x + y - 5) + \sqrt{2}(3x - 2y + 7) - 10\sqrt{26} = 0$$

Now by assuming $(x + y - 5)$ and $(3x - 2y + 7)$ are positive

$$\sqrt{13}x + \sqrt{13}y - 5\sqrt{13} + 3\sqrt{2}x - 2\sqrt{2}y + 7\sqrt{2} - 10\sqrt{26} = 0$$

Taking out the common terms

$$x(\sqrt{13} + 3\sqrt{2}) + y(\sqrt{13} - 2\sqrt{2}) + (7\sqrt{2} - 5\sqrt{13} - 10\sqrt{26}) = 0, \text{ which is the equation of a line.}$$

In the same way, we can find the equation of the line for any signs of $(x + y - 5)$ and $(3x - 2y + 7)$.

Hence, point P must move on a line.

Question 21

Find the equation of the line which is equidistant from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

Solution

It is given that

$$9x + 6y - 7 = 0 \dots\dots (1)$$

$$3x + 2y + 6 = 0 \dots\dots (2)$$

Consider P (h, k) be the arbitrary point that is equidistant from lines (1) and (2)

Here the perpendicular distance of P (h, k) from line (1) is written as

$$d_1 = \frac{|9h + 6k - 7|}{\sqrt{(9)^2 + (6)^2}} = \frac{|9h + 6k - 7|}{\sqrt{117}} = \frac{|9h + 6k - 7|}{3\sqrt{13}}$$

Similarly the perpendicular distance of P (h, k) from line (2) is written as

$$d_2 = \frac{|3h + 2k + 6|}{\sqrt{(3)^2 + (2)^2}} = \frac{|3h + 2k + 6|}{\sqrt{13}}$$

We know that P (h, k) is equidistant from lines (1) and (2) $d_1 = d_2$

Substituting the values

$$\frac{|9h + 6k - 7|}{3\sqrt{13}} = \frac{|3h + 2k + 6|}{\sqrt{13}}$$

By further calculation

$$|9h + 6k - 7| = 3|3h + 2k + 6|$$

It can be written as

$$|9h + 6k - 7| = \pm 3(3h + 2k + 6)$$

Here,

$$9h + 6k - 7 = 3(3h + 2k + 6) \text{ or } 9h + 6k - 7 = -3(3h + 2k + 6)$$

$$9h + 6k - 7 = 3(3h + 2k + 6) \text{ is not possible as}$$

$$9h + 6k - 7 = 3(3h + 2k + 6)$$

By further calculation,

$$-7 = 18 \text{ (is not correct)}$$

We know that

$$9h + 6k - 7 = -3(3h + 2k + 6)$$

By multiplication,

$$9h + 6k - 7 = -9h - 6k - 18$$

We get

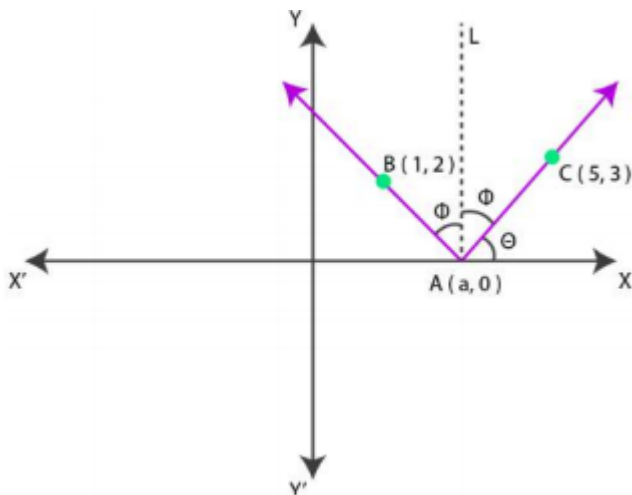
$$18h + 12k + 11 = 0$$

Hence, the required equation of the line is $18x + 12y + 11 = 0$.

Question 22

A ray of light passing through the point $(1, 2)$ reflects on the x -axis at point A , and the reflected ray passes through the point $(5, 3)$. Find the coordinates of A .

Solution



Consider the coordinates of point A as $(a, 0)$

Construct a line (AL) which is perpendicular to the x -axis

Here, the angle of incidence is equal to the angle of reflection

$$\angle BAL = \angle CAL = \phi$$

$$\angle CAX = \theta$$

It can be written as

$$\angle OAB = 180^\circ - (\theta + 2\phi) = 180^\circ - [\theta + 2(90^\circ - \theta)]$$

On further calculation,

$$= 180^\circ - \theta - 180^\circ + 2\theta$$

$$= \theta$$

So, we get

$$\angle BAX = 180^\circ - \theta$$

$$\text{slope of line AC} = \frac{3-0}{5-a}$$

$$\tan \theta = \frac{3}{5-a} \quad \dots(1)$$

$$\text{Slope of line AB} = \frac{2-0}{1-a}$$

We get

$$\tan(180^\circ - \theta) = \frac{2}{1-a}$$

By further calculation

$$-\tan \theta = \frac{2}{1-a}$$

$$\tan \theta = \frac{2}{a-1} \quad \dots(2)$$

From equations (1) and (2) we get

$$\frac{3}{5-a} = \frac{2}{a-1}$$

By cross multiplication,

$$3a - 3 = 10 - 2a$$

We get

$$a = 13/5$$

Hence, the coordinates of point A are $(13/5, 0)$.

Question 23

Prove that the product of the lengths of the perpendiculars drawn from the points

$$\left(\sqrt{a^2 - b^2}, 0\right) \text{ and } \left(-\sqrt{a^2 - b^2}, 0\right) \text{ to the line } \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \text{ is } b^2.$$

Solution

It is given that

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

We can write it as

$$bx \cos \theta + ay \sin \theta - ab = 0 \dots (1)$$

Here the length of the perpendicular from point $(\sqrt{a^2 - b^2}, 0)$ to line (1)

$$p_1 = \frac{|b \cos \theta (\sqrt{a^2 - b^2}) + a \sin \theta (0) - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} = \frac{|b \cos \theta \sqrt{a^2 - b^2} - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \dots (2)$$

Similarly the length of the perpendicular from point $(-\sqrt{a^2 - b^2}, 0)$ to line (2)

$$p_2 = \frac{|b \cos \theta (-\sqrt{a^2 - b^2}) + a \sin \theta (0) - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} = \frac{|b \cos \theta \sqrt{a^2 - b^2} + ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \dots (3)$$

By multiplying equations (2) and (3) we get

$$p_1 p_2 = \frac{|b \cos \theta \sqrt{a^2 - b^2} - ab| |b \cos \theta \sqrt{a^2 - b^2} + ab|}{(\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta})^2}$$

We get

$$= \frac{|(b \cos \theta \sqrt{a^2 - b^2} - ab)(b \cos \theta \sqrt{a^2 - b^2} + ab)|}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}$$

From the formula

$$= \frac{|(b \cos \theta \sqrt{a^2 - b^2})^2 - (ab)^2|}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}$$

By squaring the numerator we get

$$= \frac{b^2 \cos^2 \theta (a^2 - b^2) - a^2 b^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

By expanding using formula

$$= \frac{a^2 b^2 \cos^2 \theta - b^4 \cos^2 \theta - a^2 b^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

Taking out the common terms

$$= \frac{b^2 [a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2]}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

We get

$$= \frac{b^2 [a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2 \sin^2 \theta - a^2 \cos^2 \theta]}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

Here $\sin^2 \theta + \cos^2 \theta = 1$

$$= \frac{b^2 [- (b^2 \cos^2 \theta + a^2 \sin^2 \theta)]}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

So we get

$$= \frac{b^2 (b^2 \cos^2 \theta + a^2 \sin^2 \theta)}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}$$

$$= b^2$$

Therefore, it is proved.

Question 24

A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find the equation of the path that he should follow.

Solution

It is given that

$$2x - 3y + 4 = 0 \dots\dots (1)$$

$$3x + 4y - 5 = 0 \dots\dots (2)$$

$$6x - 7y + 8 = 0 \dots\dots (3)$$

Here, the person is standing at the junction of the paths represented by lines (1) and (2).

By solving equations (1) and (2), we get

$$x = -1/17 \text{ and } y = 22/17$$

Hence, the person is standing at point $(-1/17, 22/17)$.

We know that the person can reach path (3) in the least time if he walks along the perpendicular line to (3) from point $(-1/17, 22/17)$.

Here, the slope of the line (3) = $6/7$

We get the slope of the line perpendicular to the line (3) = $-1/(6/7) = -7/6$

So, the equation of the line passing through $(-1/17, 22/17)$ and having a slope of $-7/6$ is written as

$$\left(y - \frac{22}{17}\right) = -\frac{7}{6}\left(x + \frac{1}{17}\right)$$

By further calculation,

$$6(17y - 22) = -7(17x + 1)$$

By multiplication,

$$102y - 132 = -119x - 7$$

We get

$$119x + 102y = 125$$

Therefore, the path that the person should follow is $119x + 102y = 125$.
