

Chapter 9: Some Applications of Trigonometry

NCERT · Class 10 · Maths

Exercise 9.1

Question 1

A circus artist is climbing a

20 m

long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is

30°

.

Solution

Ans:

Given: A circus performer is climbing a 20 m rope that runs from the top of a vertical pole to the ground, firmly stretched and knotted.

To find: If the rope makes a

30°

angle with the ground level, calculate the height of the pole.

It can be observed from the figure that AB is the pole.

In $\triangle ABC$,

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{AB}{20} = \frac{1}{2}$$

$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

Question 2

A tree breaks due to a storm and the broken part bends so that the top of the tree touches the ground making an angle

$$30^\circ$$

with it. The distance between the feet of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solution

Ans:

Given: A storm causes a tree to shatter, and the fractured section bends such that the top of the tree meets the ground at a

$$30^\circ$$

angle. The distance from the tree's foot to where the top touches the earth is 8 meters.

To find: Determine the tree's height.

The original tree was Let AC. It was split in two due to the storm. The broken portion A'B is at a

$$30^\circ$$

angle to the ground.

In $\Delta A'BC$

$$\frac{BC}{A'C} = \tan 30^\circ$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \left(\frac{8}{\sqrt{3}} \right) \text{ m}$$

$$\frac{A'C}{A'B} = \cos 30^\circ$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \left(\frac{16}{\sqrt{3}} \right) \text{ m}$$

$$\text{Height of tree} = A'B + BC$$

$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} \right) \text{ m} = \frac{24}{\sqrt{3}} \text{ m}$$

$$= 8\sqrt{3} \text{ m}$$

Question 3

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of

$$30^\circ$$

to the ground, whereas for the elder children she wants to have a steep slide at a height of 3 m, and inclined at an angle of

$$60^\circ$$

to the ground. What should be the length of the slide in each case?

Solution

Ans:

Given: In a park, a contractor intends to construct two slides for the kids to enjoy. She likes a slide with a top height of 1.5 m and an angle of

$$30^\circ$$

to the ground for children under the age of 5, and a steep slide with a top height of 3 m and an angle of

$$60^\circ$$

to the ground for older children.

To find: In each scenario, what should the length of the slide be?

It can be noted that AC and PR are the slides for younger and elder children.

In $\triangle ABC$,

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$

In $\triangle PQR$,

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$ m.

Question 4

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is

$$30^\circ$$

. Find the height of the tower.

Solution

Ans:

Given: The angle of elevation of a tower's top from a point on the ground 30 metres distant from the tower's foot is

$$30^\circ$$

.

To find: Determine the tower's height.

Let AB be the tower and the angle of elevation from point C (on ground) is

$$30^\circ$$

. In

$$\triangle ABC$$

,

In

$$\triangle ABC$$

$$, \frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m.

Question 5

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is

$$60^\circ$$

. Find the length of the string, assuming that there is no slack in the string.

Solution

Ans:

Given: At a height of 60 m above the earth, a kite is flying. The kite's line is temporarily tethered to a location on the ground. The string is at a

$$60^\circ$$

angle to the ground.

To find: Determine the string's length, assuming there is no slack in the string.

Let K be the kite and the string is tied to point P on the ground. In

$$\triangle KLP$$

,

In $\triangle KLP$,

$$\frac{KL}{KP} = \sin 60^\circ$$

$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

Question 6

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from

$$30^\circ$$

to

$$60^\circ$$

as he walks towards the building. Find the distance he walked towards the building.

Solution

Ans:

Given: A 1.5-meter-tall child stands at a safe distance from a 30 m tall structure. As he goes towards the structure, the angle of elevation from his eyes to the top of the building grows from

$$30^\circ$$

to

$$60^\circ$$

.

To find: Calculate how far he went towards the building.

Let the boy was standing at point S initially. He walked towards the building and reached point T.

$$PR = PQ - RQ$$

$$= (30 - 1.5)\text{m} = 28.5 \text{ m} = \frac{57}{2} \text{ m}$$

In $\triangle PAR$,

$$\frac{PR}{AR} = \tan 30^\circ$$

$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$AR = \left(\frac{57}{2} \sqrt{3} \right) \text{ m}$$

In $\triangle PRB$

$$\frac{PR}{BR} = \tan 60^\circ$$

$$\frac{57}{2BR} = \sqrt{3}$$

$$BR = \frac{57}{2\sqrt{3}} = \left(\frac{19\sqrt{3}}{2} \right) \text{ m}$$

$$ST = AB$$

$$= AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2} \right) \text{ m}$$

$$= \left(\frac{38\sqrt{3}}{2} \right) \text{ m} = 19\sqrt{3} \text{ m}$$

Hence, he walked $19\sqrt{3}$ m towards the building.

Question 7

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are

$$45^\circ$$

and

$$60^\circ$$

respectively. Find the height of the tower.

Solution

Ans:

Given: The angles of elevation of the bottom and top of a transmission tower installed at the top of a 20 m high structure are

$$45^\circ$$

and

$$60^\circ$$

, respectively, from a point on the ground.

To find: Determine the tower's height.

Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.

In $\triangle BCD$,

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{20}{CD} = 1$$

$$CD = 20 \text{ m}$$

In $\triangle ACD$,

$$\frac{AC}{CD} = \tan 60^\circ$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$\frac{AB + 20}{CD} = \sqrt{3}$$

$$AB = (20\sqrt{3} - 20)\text{m}$$

$$= 20(\sqrt{3} - 1)\text{m}$$

Therefore, the height of the transmission tower is $20(\sqrt{3} - 1)\text{m}$

Question 8

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is

$$60^\circ$$

and from the same point the angle of elevation of the top of the pedestal is

$$45^\circ$$

. Find the height of the pedestal.

Solution

Ans:

Given: A 1.6 m tall statue sits on top of a pedestal; the angle of elevation of the top of the statue is

$$60^\circ$$

from a point on the ground, and the angle of elevation of the top of the pedestal is

$$45^\circ$$

from the same point.

To find: Determine the pedestal's height.

Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In $\triangle BCD$,

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{BC}{CD} = 1$$

$$BC = CD$$

In $\triangle ACD$,

$$\frac{AB + BC}{CD} = \tan 60$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$1.6 + BC = BC\sqrt{3} \quad [As CD = BC]$$

$$BC(\sqrt{3} - 1) = 1.6$$

$$BC = \frac{(1.6)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

(By Rationalization)

$$= \frac{1.6(\sqrt{3} + 1)}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{1.6(\sqrt{3} + 1)}{2} = 0.8(\sqrt{3} + 1)$$

Therefore, the height of the pedestal is $0.8(\sqrt{3} + 1)$ m

Question 9

The angle of elevation of the top of a building from the foot of the tower is

$$30^\circ$$

and the angle of elevation of the top of the tower from the foot of the building is

$$60^\circ$$

. If the tower is 50 m high, find the height of the building.

Solution

Answer :

Given: The angle of elevation of a building's top from the foot of the tower is

$$30^\circ$$

, while the angle of elevation of a tower from the foot of the building is

$$60^\circ$$

.

To find: Find the height of the building if the tower is 50 m tall.

Let AB be the building and CD be the tower.

In $\triangle CDB$,

$$\frac{CD}{BD} = \tan 60^\circ$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

Question 10

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are

$$60^\circ$$

and

$$30^\circ$$

, respectively. Find the height of poles and the distance of the point from the poles.

Solution

Ans:

Given: On either side of the 80 m wide road, two poles of equal height stand opposite each other. The angles of elevation of the tops of the poles are

$$60^\circ$$

and

$$30^\circ$$

, respectively, from a point on the route between them.

To find: Determine the height of the poles and the distance between them.

Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In $\triangle ABO$,

$$\frac{AB}{BO} = \tan 60$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In $\triangle CDO$,

$$\frac{CD}{DO} = \tan 30^\circ$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights, $CD = AB$

$$CD \left[\sqrt{3} + \frac{1}{\sqrt{3}} \right] = 80$$

$$CD \left(\frac{3+1}{\sqrt{3}} \right) = 80$$

$$CD = 20\sqrt{3} \text{ m}$$

$$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = \left(\frac{20\sqrt{3}}{\sqrt{3}} \right) \text{ m} = 20 \text{ m}$$

$$DO = BD - BO = (80 - 20) \text{ m} = 60 \text{ m}$$

Therefore, the height of poles is $20\sqrt{3}$ and the point is 20 m and 60 m far from the D

Question 11

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is

$$60^\circ$$

. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is

$$30^\circ$$

. Find the height of the tower and the width of the canal.

Solution

Ans:

Given: On the edge of a canal, a TV tower stands vertically. The angle of elevation of the top of the tower from a location on the other bank exactly opposite the tower is

$$60^\circ$$

. The angle of elevation of the top of the tower is

$$30^\circ$$

measured from another point 20 m distant on the line connecting this point to the foot of the tower.

To find: Calculate the tower's height and the canal's width.

In $\triangle ABC$

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

$$AB = 10\sqrt{3} \text{ m}$$

$$BC = \frac{AB}{\sqrt{3}} = \left(\frac{10\sqrt{3}}{\sqrt{3}} \right) \text{ m} = 10 \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

Question 12

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is

$$60^\circ$$

and the angle of depression of its foot is

$$45^\circ$$

. Determine the height of the tower.

Solution

Ans:

Given: From the top of a 7 m high building, the angle of elevation of the top of a cable tower is

$$60^\circ$$

and the angle of depression of its foot is

$$45^\circ$$

.

To find: Determine the height of the tower.

Let AB be a building and CD be a cable tower. In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7}{BD} = 1 \Rightarrow BD = 7 \text{ m}$$

In $\triangle ACE$,

$$AE = BD = 7 \text{ m}$$

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{CE}{7} = \sqrt{3}$$

$$CE = 7\sqrt{3} \text{ m}$$

$$CD = CE + ED = (7\sqrt{3} + 7)\text{m}$$

$$= 7(\sqrt{3} + 1)\text{m}$$

Therefore, the height of the cable tower is $(7\sqrt{3} + 7)$ m.

Question 13

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are

$$30^\circ$$

and

$$45^\circ$$

. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Solution

Ans:

Given: The angles of depression of two ships are

$$30^\circ$$

and

$$45^\circ$$

, respectively, as seen from the top of a 75 m high lighthouse from sea level.

To find: Find the distance between the two ships if one is precisely behind the other on the same side of the lighthouse.

Let the distance between the ships be x m.

Then in right $\triangle ADB$, $\tan 30^\circ = \frac{AB}{BD}$

$$\Rightarrow 1 = \frac{75}{y} \Rightarrow y = 75 \text{ m}$$

In right $\triangle ADB$, $\tan 30^\circ = \frac{AB}{BD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x + y} \Rightarrow x + 75 = 75\sqrt{3} \quad [\text{From equation (i)}]$$

$$\Rightarrow x = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1)$$

Hence, the distance between the two ships is $75(\sqrt{3} - 1)$ m

Question 14

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is

$$60^\circ$$

. After some time, the angle of elevation reduces to

$$30^\circ$$

. Find the distance travelled by the balloon during the interval.

Solution

Ans:

Given: A 1.2 m tall girl notices a balloon floating horizontally with the wind at a height of 88.2 m above the earth. At any given time, the angle of elevation of the balloon from the girl's eyes is

$$60^\circ$$

. After a while, the angle of elevation drops to

$$30^\circ$$

To find: Calculate the balloon's distance travelled throughout the timeframe.

Given that 1.2 m tall girl sees a balloon

So, $AG = 1.2$ m

Also, AG & BF are parallel $BF = AG = 1.2$ m

And the balloon is at a height of 88.2 m

So, $EF = 88.2$

Girl sees balloon first at 60°

So, $\angle EAB = 60^\circ$

After travelling, the angle of elevation becomes 30°

$$\text{So, } \angle DAC = 30^\circ$$

Distance travelled by the balloon = BC

We have to find BC

Now,

$$BE = EF - BF$$

$$BE = 88.2 - 1.2 \text{ m}$$

$$BE = 87 \text{ m}$$

$$\text{Also, } BE = DC = 87 \text{ m}$$

Here,

$$\angle ABE = 90^\circ \text{ \& } \angle ACD = 90^\circ$$

In right angle triangle EBA

$$\tan A = \frac{\text{Side opposite to angle A}}{\text{Side adjacent to angle A}}$$

$$\tan A = \frac{BE}{AB}$$

$$\tan 60^\circ = \frac{BE}{AB}$$

$$\sqrt{3} = \frac{87}{AB}$$

$$AB = \frac{87}{\sqrt{3}} \text{ m}$$

In right angle triangle DAC

$$\tan A = \frac{\text{Side opposite to angle A}}{\text{Side adjacent to angle A}}$$

$$\tan A = \frac{CD}{AC}$$

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{87}{AC}$$

$$AC = 87\sqrt{3} \text{ m}$$

$$AC = AB + BC$$

$$87\sqrt{3} = \frac{87}{\sqrt{3}} + BC$$

$$87\sqrt{3} - \frac{87}{\sqrt{3}} = BC$$

$$BC = 87\sqrt{3} - \frac{87}{\sqrt{3}}$$

$$BC = 87 \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right)$$

$$BC = 87 \left(\frac{\sqrt{3}\sqrt{3} - 1}{\sqrt{3}} \right)$$

$$BC = 87 \left(\frac{3 - 1}{\sqrt{3}} \right)$$

$$BC = 87 \left(\frac{2}{\sqrt{3}} \right)$$

$$BC = \frac{87 \times 2}{\sqrt{3}}$$

Multiply $\sqrt{3}$ in numerator and denominator

$$BC = \frac{87 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$BC = \frac{87 \times 2 \times \sqrt{3}}{3}$$

$$BC = 29 \times 2 \times \sqrt{3}$$

$$BC = 58\sqrt{3}$$

Hence, distance travelled by balloon = $58\sqrt{3}$ m

Question 15

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of

$$30^\circ$$

, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be

$$60^\circ$$

. Find the time taken by the car to reach the foot of the tower from this point.

Solution

Answer:

Let the height of the tower DC be h m and the speed of the car be x m/s. Then the distance covered by the car in 6 s will be $6x$ m.

Also let the time taken by car to move from B to C be t s.

Then the distance BC will be xt m

In right $\triangle DCB$,

$$\tan 60^\circ = \frac{DC}{BC} \Rightarrow \sqrt{3} = \frac{h}{tx}$$

$$\Rightarrow h = \sqrt{3}tx$$

In right $\triangle ACD$,

$$\tan 30^\circ = \frac{DC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{6x + tx}$$

$$\Rightarrow 6x + tx = \sqrt{3}h$$

$$\Rightarrow x(6 + t) = \sqrt{3} \times \sqrt{3}tx$$

$$\Rightarrow 6 + t = 3t \Rightarrow 2t = 6$$

$\Rightarrow$ [From (i)]

$$\Rightarrow t = 3 \text{ s}$$

Hence, the required time taken by car is **3 s**.
