

Chapter 7: Coordinate Geometry

NCERT · Class 10 · Maths

Exercise 7.1

Question 1

Find the distance between the following pairs of points:

(i)

$$(2,3), (4,1)$$

(ii)

$$(-5,7), (-1,3)$$

(iii)

$$(a,b), (-a,-b)$$

Solution

Ans: The given points are

$$A(2,3)$$

and

$$B(4,1)$$

Using the Distance Formula =

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

By comparing the points with the formula we get,

$$x_1=2$$

,

$$y_1=3$$

,

$$x_2=4$$

,

$$y_2=1$$

Distance between

$$A (2,3)$$

and

$$B (4,1)$$

is given by,

$$AB =$$

$$\sqrt{(2-4)^2+(3-1)^2}$$

$$=$$

$$\sqrt{(-2)^2+(2)^2}$$

$$=$$

$$\sqrt{4+4}$$

$$=$$

$$\sqrt{8}$$

$$=$$

$$2\sqrt{2}$$

Ans: The given points are

$$C (-5, 7)$$

and

$$D (-1, 3)$$

Using the Distance Formula =

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

By comparing the points with the formula we get,

$$x_1 = -5$$

,

$$y_1 = 7$$

,

$$x_2 = -1$$

,

$$y_2 = 3$$

Distance between

$$C (-5,7)$$

and

$$D (-1,3)$$

is given by,

$$CD =$$

$$\sqrt{(-5-(-1))^2+(7-3)^2}$$

=

$$\sqrt{(-4)^2+(4)^2}$$

=

$$\sqrt{16+16}$$

=

$$\sqrt{32}$$

=

$$4\sqrt{2}$$

Ans: The given points are

$$X(a, b)$$

and

$$Y(-a, -b)$$

Using the Distance Formula =

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

By comparing the points with the formula we get,

$$x_1 = a$$

,

$$y_1 = b$$

,

$$x_2 = -a$$

,

$$y_2 = -b$$

Distance between

$$X(a, b)$$

and

$$Y(-a, -b)$$

is given by,

$$XY =$$

$$\sqrt{(a-(-a))^2 + (b-(-b))^2}$$

$$=$$

$$\sqrt{(2a)^2 + (2b)^2}$$

$$=$$

$$\sqrt{4a^2 + 4b^2}$$

$$=$$

$$2\sqrt{a^2 + b^2}$$

Question 2

Find the distance between the points

$$(0,0)$$

and

$$(36,15)$$

. Can you now find the distance between the two towns A and B discussed in Section 7.2?

Solution

Ans: According to the , the given points are

$$P (0,0)$$

and

$$Q (36,15)$$

.

Using the Distance Formula =

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

By comparing the points with the formula we get,

$$x_1=0$$

,

$$y_1=0$$

,

$$x_2=36$$

,

$$y_2=15$$

.

The distance between the points

$$P (0,0)$$

and

$$Q (36,15)$$

is given by,

PQ =

$$\sqrt{(36-0)^2 + (15-0)^2}$$

=

$$\sqrt{36^2 + 15^2}$$

=

$$\sqrt{1296 + 225}$$

=

$$\sqrt{1521}$$

=

$$39$$

Yes, we can find the distance between two towns A and B using the distance formula only. Two towns are situated at point (0,0) and (36,15) respectively. So, the distance between both towns will be 39 km.

Question 3

Determine if the points

$$(1,5), (2,3)$$

and

$$(-2,-11)$$

are collinear.

Solution

Ans: Three points are collinear if and only if they are on the same line.

Let the given points are

$$A(1,5), B(2,3)$$

and

$$C(-2, -11)$$

Using the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

,

$$AB = \sqrt{(1 - 2)^2 + (5 - 3)^2}$$

=

$$\sqrt{1^2 + 2^2}$$

=

$$\sqrt{5}$$

$$BC = \sqrt{(2 - (-2))^2 + (3 - (-11))^2}$$

=

$$\sqrt{16 + 196}$$

=

$$\sqrt{212}$$

$$AC = \sqrt{(1 - (-2))^2 + (5 - (-11))^2}$$

=

$$\sqrt{9 + 256}$$

=

$$\sqrt{265}$$

Here,

$$AB + AC \neq BC$$

and

$$BC + AC \neq AB$$

and

$$AB + BC \neq AC$$

. Since, the sum of distance of any two point is not equal to third point.

Therefore, the points

$$(1,5), (2,3)$$

and

$$(-2,-11)$$

are not collinear.

Question 4

Check whether

$$(5,-2), (6,4)$$

and

$$(7,-2)$$

are the vertices of an isosceles triangle.

Solution

Ans: To prove the points to be an isosceles triangle the points must be collinear and any two sides must be of equal length.

Let the points

$$(5,-2), (6,4)$$

and

$$(7,-2)$$

are representing the vertices

A, B, and C of the given triangle respectively.

Using the Distance Formula =

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

,

$$AB = \sqrt{(5-6)^2+(-2-4)^2}$$

=

$$\sqrt{(-1)^2+(-6)^2}$$

=

$$\sqrt{1+36}$$

=

$$\sqrt{37}$$

$$BC = \sqrt{(6-7)^2+(4-(-2))^2}$$

=

$$\sqrt{(-1)^2+(6)^2}$$

=

$$\sqrt{1+36}$$

=

$$\sqrt{37}$$

$$AC = \sqrt{(5-7)^2+(-2-(-2))^2}$$

=

$$\sqrt{(-2)^2 + (0)^2}$$

=

$$\sqrt{4}$$

= 2

Here, $AB = BC$. As two sides are equal in length and the sum of distance of any two point is not equal to third point. Therefore, ABC is an isosceles triangle.

Question 5

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, “Don’t you think ABCD is a square?” Chameli disagrees. Using distance formula, find which of them is correct.

Solution

Ans: From the above figure, the coordinates of A, B, C, D are

$$(3,4), (6,7), (9,4)$$

and

$$(6,1)$$

respectively.

Using the Distance formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$AB = \sqrt{(3 - 6)^2 + (4 - 7)^2}$$

=

$$\sqrt{(-3)^2 + (-3)^2}$$

=

$$\sqrt{9+9}$$

=

$$\sqrt{18}$$

=

$$3\sqrt{2}$$

$$BC = \sqrt{(6-9)^2 + (7-4)^2}$$

=

$$\sqrt{(-3)^2 + 3^2}$$

=

$$\sqrt{9+9}$$

=

$$\sqrt{18}$$

=

$$3\sqrt{2}$$

$$CD = \sqrt{(9-6)^2 + (4-1)^2}$$

=

$$\sqrt{3^2 + 3^2}$$

=

$$\sqrt{9+9}$$

=

$$\sqrt{18}$$

=

$$3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2}$$

=

$$\sqrt{(-3)^2 + 3^2}$$

=

$$\sqrt{9+9}$$

=

$$\sqrt{18}$$

=

$$3\sqrt{2}$$

Diagonal

$$AC = \sqrt{(3-9)^2 + (4-4)^2}$$

=

$$\sqrt{(-6)^2 + 0^2}$$

=

$$6$$

Diagonal

$$BD = \sqrt{(6-6)^2 + (7-1)^2}$$

=

$$\sqrt{0^2 + 6^2}$$

=

6

Here, all four sides AB, BC, CD, and AD are of same length and diagonals AC and BD are also of equal length. Therefore, we can conclude that ABCD is a square and hence, Champa was correct.

Question 6

Name the type of quadrilateral formed, if any, by the following points, and

give reasons for your Ans:

i.

$$(-1, -2), (1, 0), (-1, 2), (-3, 0)$$

ii.

$$(-3, 5), (3, 1), (0, 3), (-1, -4)$$

iii.

$$(4, 5), (7, 6), (4, 3), (1, 2)$$

Solution

Ans: Let the points

$$(-1, -2), (1, 0), (-1, 2), (-3, 0)$$

be representing the vertices A, B, C, and D of the given quadrilateral respectively.

Using the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

,

$$AB = \sqrt{(-1 - 1)^2 + (-2 - 0)^2}$$

$$= \sqrt{(-2)^2 + (-2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

=

$$2\sqrt{2}$$

$$BC = \sqrt{(1 - (-1))^2 + (0 - 2)^2}$$

$$= \sqrt{(2)^2 + (-2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

=

$$2\sqrt{2}$$

$$CD = \sqrt{(-1 - (-3))^2 + (2 - 0)^2}$$

$$= \sqrt{2^2 + 2^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

=

$$2\sqrt{2}$$

$$AD = \sqrt{(-1 - (-3))^2 + (-2 - 0)^2}$$

$$= \sqrt{2^2 + 2^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

=

$$2\sqrt{2}$$

Diagonal

$$AC = \sqrt{(-1 - (-1))^2 + ((-2) - 2)^2}$$

$$= \sqrt{0+4^2}$$

$$= \sqrt{16}$$

$$= 4$$

Diagonal

$$BD = \sqrt{(1 - (-3))^2 + (0 - 0)^2}$$

$$= \sqrt{4^2 + 0}$$

$$= \sqrt{16}$$

$$= 4$$

Therefore, $AB = BC = CD = AD$

Here, all the sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

Ans: Let the points

$$(-3,5), (3,1), (0,3), (-1,-4)$$

be representing the vertices A, B, C, and D of the given quadrilateral respectively.

Using the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

,

$$AB = \sqrt{(-3 - 3)^2 + (5 - 1)^2}$$

$$= \sqrt{(-6)^2 + (4)^2}$$

$$= \sqrt{36 + 16}$$

$$= \sqrt{52}$$

$$= 2\sqrt{13}$$

$$BC = \sqrt{(3 - 0)^2 + (1 - 3)^2}$$

$$= \sqrt{3^2 + (-2)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

$$CD = \sqrt{(0 - (-1))^2 + (3 - (-4))^2}$$

$$= \sqrt{1^2 + 7^2}$$

$$= \sqrt{1+49}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2}$$

$$AD = \sqrt{(-3 - (-1))^2 + (5 - (-4))^2}$$

$$= \sqrt{(-2)^2 + 9^2}$$

$$= \sqrt{4+81}$$

$$= \sqrt{85}$$

Diagonal

$$AC = \sqrt{(0 - (-3))^2 + (3 - 5)^2}$$

$$= \sqrt{3^2 + (-2)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

Diagonal

$$BD = \sqrt{(-1 - 3)^2 + (-4 - 1)^2}$$

$$= \sqrt{(-4)^2 + (-5)^2}$$

$$= \sqrt{16 + 25}$$

$$= \sqrt{41}$$

Here, all the sides of this quadrilateral are of different length i.e.,

$$AB \neq BC \neq AC \neq AD$$

and also $AC + BC = AB$ which means that A, B, C are collinear. Therefore, no quadrilateral can be formed from these points.

Ans: Let the points

$$(4,5), (7,6), (4,3), (1,2)$$

be representing the vertices A, B, C, and D of the given quadrilateral respectively.

Using the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

,

$$AB = \sqrt{(4 - 7)^2 + (5 - 6)^2}$$

$$= \sqrt{(-3)^2 + (-1)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2}$$

$$= \sqrt{3^2 + 3^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2}$$

$$= \sqrt{3^2 + 1^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10}$$

$$AD = \sqrt{(4-1)^2 + (5-2)^2}$$

$$= \sqrt{3^2 + 3^2}$$

$$\begin{aligned} &= \sqrt{9+9} \\ &= \sqrt{18} \end{aligned}$$

Diagonal

$$\begin{aligned} AC &= \sqrt{(4-4)^2 + (5-3)^2} \\ &= \sqrt{0+2^2} \\ &= \sqrt{4} = 2 \end{aligned}$$

Diagonal

$$\begin{aligned} BD &= \sqrt{(7-1)^2 + (6-2)^2} \\ &= \sqrt{6^2 + 4^2} \\ &= \sqrt{36+16} = \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

Here, the opposite sides of this quadrilateral are of the same length i.e., $AB = CD$ and $BC = AD$. But, the diagonals are of different lengths.

Therefore, the given points are the vertices of a parallelogram.

Question 7

Find the point on the x-axis which is equidistant from

$$(2, -5)$$

and

$$(-2,9)$$

.

Solution

Ans: If the point is on x-axis the co-ordinates will be

$$P(x, 0)$$

. We have to find a point on x-axis which is equidistant from

$$A(2, -5)$$

and

$$B(-2, 9)$$

.

Using the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$PA = \sqrt{(x - 2)^2 + (0 - (-5))^2}$$

$$= \sqrt{(x - 2)^2 + 5^2}$$

$$PB = \sqrt{(x - (-2))^2 + (0 - (-9))^2}$$

$$= \sqrt{(x + 2)^2 + 9^2}$$

By the given condition, these distances are equal in measure.

Hence $PA = PB$

$$\sqrt{(x - 2)^2 + 5^2} = \sqrt{(x + 2)^2 + 9^2}$$

$$\Rightarrow (x - 2)^2 + 25 = (x + 2)^2 + 81$$

$$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$\Rightarrow 8x = 25 - 81$$

$$\Rightarrow 8x = -56$$

$$\Rightarrow x = -7$$

Hence,

$$P(-7, 0)$$

is the point on the x-axis which is equidistance from the given points.

Question 8

Find the values of y for which the distance between the points

$$P(2, -3)$$

and

$$Q(10, y)$$

is

$$10$$

units.

Solution

Ans: We know that the distance between the two points is given by the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance between points

$$A(2, -3)$$

and

$$B(10,y)$$

is

$$10$$

units. Therefore, by substituting the values of points A (2, -3) and B (10, y) in Distance Equation

$$\sqrt{(2-10)^2 + (-3-y)^2} = 10$$

$$\Rightarrow \sqrt{(-8)^2 + (3+y)^2} = 10$$

Squaring both sides

$$\Rightarrow 64 + (3+y)^2 = 100$$

$$\Rightarrow (y+3)^2 = 36$$

$$\Rightarrow (y+3) = \pm 6$$

$$\Rightarrow (y+3) = 6$$

or

$$(y+3) = -6$$

Therefore,

$$y = -3 \text{ or } 9$$

are the possible values for y.

Question 9

If

$$Q(0,1)$$

is equidistant from

$$P(5,-3)$$

and

$$R(x,6)$$

, find the values of x . Also find the distance QR and PR.

Solution

Ans: We know that the distance between the two points is given by the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

As

$$Q(0,1)$$

is equidistant from

$$P(5,-3)$$

and

$$R(x,6)$$

.

So, $PQ = QR$

$$\begin{aligned}\sqrt{(5-0)^2 + (-3-1)^2} &= \sqrt{(0-x)^2 + (1-6)^2} \\ \Rightarrow \sqrt{5^2 + (-4)^2} &= \sqrt{(-x)^2 + (-5)^2} \\ \Rightarrow 25 + 16 &= x^2 + 25\end{aligned}$$

(By squaring both sides)

$$\begin{aligned}\Rightarrow 16 &= x^2 \\ \Rightarrow x^2 &= \pm 4\end{aligned}$$

Therefore, point R is

$$(4,6)$$

or

$$(-4,6)$$

Case (1),

When point R is

$$(4,6)$$

,

Distance between

$$P(5,-3)$$

and

$$R(4,6)$$

PR=

$$\sqrt{(5-4)^2 + (-3-6)^2}$$

$$= \sqrt{(1)^2 + (-9)^2}$$

$$= \sqrt{1+81}$$

$$= \sqrt{82}$$

Distance between

$$Q(0,1)$$

and

$$R(4,6)$$

QR =

$$\sqrt{(0-4)^2 + (1-6)^2}$$

$$= \sqrt{(-4)^2 + (-5)^2}$$

$$= \sqrt{16 + 25}$$

$$= \sqrt{41}$$

Case (2),

When point R is

$$(-4,6)$$

,

Distance between

$$P(5,-3)$$

and

$$R(-4,6)$$

PR =

$$\sqrt{(5-(-4))^2 + (-3-6)^2}$$

$$= \sqrt{(9)^2 + (-9)^2}$$

$$= \sqrt{81 + 81}$$

$$= 9\sqrt{2}$$

Distance between

$$Q(0,1)$$

and

$$R(-4,6)$$

QR =

$$\sqrt{(0 - (-4))^2 + (1 - 6)^2}$$

$$= \sqrt{(4)^2 + (-5)^2}$$

$$= \sqrt{16 + 25}$$

$$= \sqrt{41}$$

Question 10

Find a relation between x and y such that the point (x, y) is equidistant from the point

$$(3,6)$$

and

$$(-3,4)$$

.

Solution

Ans: We know that the distance between the two points is given by the Distance Formula =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Point P(x, y) be equidistant from points

$$A(3,6)$$

and

$$B(-3,4)$$

.

Therefore, PA = PB

$$\begin{aligned}\sqrt{(x-3)^2 + (y-6)^2} &= \sqrt{(x-(-3))^2 + (y-4)^2} \\ \Rightarrow \sqrt{(x-3)^2 + (y-6)^2} &= \sqrt{(x+3)^2 + (y-4)^2}\end{aligned}$$

Squaring both sides

$$\begin{aligned}\Rightarrow (x-3)^2 + (y-6)^2 &= (x+3)^2 + (y-4)^2 \\ \Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y &= x^2 + 9 + 6x + y^2 + 16 - 8y \\ \Rightarrow 36 - 16 &= 6x + 6x + 12y - 8y \\ \Rightarrow 20 &= 12x + 4y \\ \Rightarrow 3x + y &= 5 \\ \Rightarrow 3x + y - 5 &= 0\end{aligned}$$

Thus, the relation between x and y is given by

$$3x + y - 5 = 0$$

Exercise 7.2

Question 1

Find the coordinates of the point which divides the join of

$$(-1, 7)$$

and

$$(4, -3)$$

in the ratio

$$2:3$$

Solution

Ans: Let P

$$(x, y)$$

be the required point.

Let A

$$(-1, 7)$$

and B

$$(4, -3)$$

where

$$x_1 = -1, y_1 = 7, x_2 = 4, y_2 = -3$$

and

$$m:n = 2:3$$

So, by section formula

$$P(x,y)$$

=

$$\left[\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right]$$

Now, substituting the values we get,

$$x = \left[\frac{2(4) + 3(-1)}{2 + 3} \right] = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

$$y = \left[\frac{2(-3) + 3(7)}{2 + 3} \right] = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Therefore, the co-ordinates of point P are

$$(1,3)$$

Question 2

Find the coordinates of the points of trisection of the line segment joining

$$(4, -1)$$

and

$$(-2, -3)$$

Solution

Ans:

Let line segment joining the points be A

$$(4, -1)$$

and B

$$(-2, -3)$$

.

Let P

$$(x_1, y_1)$$

and Q

$$(x_2, y_2)$$

are the points of trisection of the line segment joining the given points i.e., $AP = PQ = QB$

So, by section formula

$$P(x, y)$$

=

$$\left[\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right]$$

Therefore, by observation point P divides AB internally in the ratio

$$1:2$$

Now, substituting the values we get,

$$x_1 = \left[\frac{1(-2) + 2(4)}{1 + 2} \right] = \frac{-2 + 8}{3} = \frac{6}{3} = 2$$

$$y_1 = \left[\frac{1(-3) + 2(-1)}{1 + 2} \right] = \frac{-3 - 2}{3} = \frac{-5}{3}$$

Therefore, P

$$(x_1, y_1)$$

=

$$\left(2, -\frac{5}{3} \right)$$

Therefore, by observation point Q divides AB internally in the ratio

$$2:1$$

Now, substituting the values we get,

$$x_1 = \left[\frac{2(-2) + 1(4)}{2 + 1} \right] = \frac{-4 + 4}{3} = \frac{0}{3} = 0$$

$$y_1 = \left[\frac{2(-3) + 1(-1)}{2 + 1} \right] = \frac{-6 - 1}{3} = \frac{-7}{3}$$

Therefore, Q

$$(x_2, y_2)$$

=

$$(0, -\frac{7}{3})$$

Question 3

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of

1 m

each.

100

flower pots have been placed at a distance of

1 m

from each other along AD, as shown in the following figure. Niharika runs

$\frac{1}{4}$ th

the distance AD on the 2nd line and posts a green flag. Preet runs

$\frac{1}{5}$ th

the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segments joining the two flags, where should she post her flag?

Solution

Ans: By observation, that Niharika posted the green flag at of the distance P i.e.,

$$\left(\frac{1}{4} \times 100\right) \text{ m} = 25 \text{ m}$$

from the starting point of 2nd line. Therefore, the coordinates of this point P is

$$(2, 25)$$

Similarly, Preet posted red flag at of the distance Q i.e.,

$$\left(\frac{1}{5} \times 100\right) \text{ m} = 20 \text{ m}$$

from the starting point of 8th line. Therefore, the coordinates of this point Q are

$$(8, 20)$$

We know that the distance between the two points is given by the distance formula

i.e.

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

To find the distance between these flags PQ by substituting the values we get,

PQ =

$$\sqrt{(8 - 2)^2 + (25 - 20)^2} = \sqrt{36 + 25} = \sqrt{61} \text{ m}$$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be M

$$(x, y)$$

So, by section formula

M

$$(x,y)$$

=

$$\left[\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right]$$

Now, substituting the values we get,

$$x = \frac{2 + 8}{2} = \frac{10}{2} = 5$$

$$y = \frac{25 + 20}{2} = \frac{45}{2} = 22.5$$

Therefore, Rashmi should post her blue flag at

$$22.5 \text{ m}$$

on fifth line.

Question 4

Find the ratio in which the line segment joining the points

$$(-3, 10)$$

and

$$(6, -8)$$

is divided by

$$(-1, 6)$$

.

Solution

Ans:

Let the ratio in which the line segment joining A

$$(-3, 10)$$

and B

$$(6, -8)$$

is divided by point P

$$(-1, 6)$$

be

$$k:1$$

.

So, by section formula

M

$$(x, y)$$

=

$$\left[\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right]$$

$$\therefore -1 = \frac{6k - 3}{k + 1}$$

$$-k - 1 = 6k - 3$$

$$7k = 2$$

(by cross multiplying and transposing)

$$k = \frac{2}{7}$$

Hence the point P divides AB in the ratio

$$2 : 7$$

Question 5

Find the ratio in which the line segment joining A

$$(1, -5)$$

and B

$$(-4, 5)$$

is divided by the x-axis. Also find the coordinates of the point of division.

Solution

Ans:

Let the ratio be

$$k:1$$

and let the line segment joining A

$$(1, -5)$$

and B

$$(-4, 5)$$

So, by section formula

M

$$(x, y)$$

=

$$\left[\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right]$$

Now, by substituting the values we get the coordinate of the point of division

$$\left[\frac{-4k + 1}{k + 1}, \frac{5k - 5}{k + 1} \right]$$

We know that y-coordinate of any point on x-axis is

$$0$$

$$\therefore \frac{5k - 5}{k + 1} = 0 \Rightarrow 5k - 5 = 0 \Rightarrow 5k = 5 \Rightarrow k = 1$$

Therefore, x-axis divides it in the ratio

$$1 : 1$$

Division point =

$$\left(\frac{-4(1) + 1}{1 + 1}, \frac{5(1) + 5}{1 + 1} \right) = \left(\frac{-4 + 1}{2}, \frac{5 + 5}{2} \right) = \left(\frac{-3}{2}, 0 \right)$$

Question 6

If

$$(1, 2)$$

,

$$(4, y)$$

,

$$(x, 6)$$

and

$$(3, 5)$$

are the vertices of a parallelogram taken in order, find

$$x$$

and

$$y$$

Solution

Ans:

Let A

$$(1,2)$$

, B

$$(4, y)$$

, C

$$(x,6)$$

and D

$$(3,5)$$

are the vertices of a parallelogram ABCD. Since the diagonals of a parallelogram bisect each other, Intersection point O of diagonal AC and BD also divides these diagonals Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\frac{1+x}{2} = \frac{7}{2} \text{ and } \frac{5+y}{2} = 4$$

$$x + 1 = 7 \text{ and } 5 + y = 8$$

$$x = 6 \text{ and } y = 3$$

Question 7

Find the coordinates of a point A, where AB is the diameter of circle whose centre is

$$(2, -3)$$

and B is

$$(1,4)$$

Solution

Ans:

Let the coordinates of point A be

$$(x,y)$$

Mid-point of AB is C

$$(2, -3)$$

, which is the center of the circle.

$$(2, -3) = \left(\frac{x+1}{2}, \frac{y+4}{2} \right)$$

$$\frac{x+1}{2} = 2 \text{ and } \frac{y+4}{2} = -3$$

$$x + 1 = 4 \text{ and } y + 4 = -6$$

$$x = 3 \text{ and } y = -10$$

.

So, coordinates of A are

$$(3, -10)$$

Question 8

If A and B are

$$(-2, -2)$$

and

$$(2, -4)$$

respectively, find the coordinates of P such that

$$AP = \frac{3}{7}AB$$

and P lies on the line segment AB.

Solution

Ans:

The coordinates of point A and B are

$$(-2, -2)$$

and

$$(2, -4)$$

respectively and

$$AP = \frac{3}{7}AB$$

so,

$$\frac{AP}{AB} = \frac{3}{7}$$

We know that $AB = AP + PB$ from figure,

$$\frac{AP + PB}{AP} = \frac{3 + 4}{3}$$

$$1 + \frac{PB}{AP} = 1 + \frac{4}{3}$$

$$\frac{PB}{AP} = \frac{4}{3}$$

Therefore,

$$PB:AP = 4:3$$

Point P

$$(x,y)$$

divides the line segment AB in the ratio

$$3:4$$

Coordinates of P

$$(x,y)$$

=

$$\left(\frac{3 * 2 + 4 * (-2)}{3 + 4}, \frac{3 * (-4) + 4 * (-2)}{3 + 4} \right)$$

=

$$\left(\frac{6 - 8}{7}, \frac{-12 - 8}{7} \right) = \left(\frac{-2}{7}, \frac{-20}{7} \right)$$

Question 9

Find the coordinates of the points which divide the line segment joining A

$$(-2, 2)$$

and B

$$(2, 8)$$

into four equal parts.

Solution

Ans:

By observation,

$$P_1$$

,

$$P_2$$

,

$$P_3$$

that points divides the line segment A

$$(-2, 2)$$

and B

$$(2, 8)$$

into four equal parts. Point

$$P_1$$

divides the line segment A

$$P_2$$

into two equal parts

Hence, Coordinates of

$$P_1$$

=

$$\left(\frac{1 \times 2 + 3 \times (-2)}{1 + 3}, \frac{1 \times 8 + 3 \times 2}{1 + 3} \right) = \left(-1, \frac{7}{2} \right)$$

Point

$$P_2$$

divides the line segment AB into two equal parts

Coordinates of

$$P_2$$

=

$$\left(\frac{2 + (-2)}{2}, \frac{2 + 8}{2} \right) = (0, 5)$$

Point

$$P_3$$

divides the line segment B

$$P_2$$

into two equal parts

Coordinates of

$$P_3$$

=

$$\left(\frac{3 \times 2 + 1 \times (-2)}{1 + 3}, \frac{3 \times 8 + 1 \times 2}{1 + 3} \right) = \left(1, \frac{13}{2} \right)$$

Question 10

Find the area of a rhombus if its vertices are

$$(3, 0)$$

,

$$(4, 5)$$

,

$$(-1, 4)$$

and

$$(-2, -1)$$

taken in order.

[Hint: Area of a rhombus = (product of its diagonals)]

Solution

Ans:

Let A

$$(3,0)$$

,B

$$(4,5)$$

,C

$$(-1,4)$$

and D

$$(-2, -1)$$

are the vertices of a rhombus ABCD.

We know that the distance between the two points is given by the distance formula

i.e.

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore, distance between A

$$(3,0)$$

and C

$$(-1,4)$$

is given by

Length of diagonal AC =

$$\sqrt{[3 - (-1)]^2 + (0 - 4)^2} = \sqrt{16 + 16} = 4\sqrt{2}$$

Therefore, distance between B

$$(4, 5)$$

and D

$$(-2, -1)$$

is given by

Length of diagonal BD =

$$\sqrt{[4 - (-2)]^2 + (5 - (-1))^2} = \sqrt{36 + 36} = 6\sqrt{2}$$

Area of rhombus ABCD =

$$\frac{1}{2} \times (\text{product of lengths of diagonals})$$

=

$$\frac{1}{2} AC \times BD$$

=

$$\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24 \text{ square units}$$