

## Chapter 6: Triangles

NCERT · Class 10 · Maths

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### Exercise 6.1

#### Question 1

Fill in the Blanks Using Correct Word Given in the Brackets:

- i. All circles are \_\_\_\_\_. (congruent, similar)
- ii. All squares are \_\_\_\_\_. (similar, congruent)
- iii. All \_\_\_\_\_ triangles are similar. (isosceles, equilateral)
- iv. Two polygons of the same number of sides are similar, if their corresponding angles are \_\_\_\_\_ (equal, proportional)

#### Solution

Ans: Similar.

Ans: Similar.

Ans: Equilateral.

Ans: Equal

And their corresponding sides are \_\_\_\_\_. (equal, proportional)

Ans: Proportional.

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#### Question 2

Give Two Different Examples of Pair of -

**i. Similar Figures****ii. Non-Similar Figures****Solution**

Ans: The two examples for similar figures are

1. Two equilateral triangles having sides

2cm

and 4cm.

1. Two squares having sides

2cm

and 4cm.

Ans: The two examples for non - similar figures are

1. A circle and a square.

1. A triangle and a rectangle.

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**Question 3**

**State whether the following quadrilaterals are similar or not.**

### Solution

Ans: The given quadrilaterals  $PQRS$  and  $ABCD$  are not similar because their corresponding sides are proportional, that is,  $1 : 2$  but their corresponding angles are not equal.

## Exercise 6.2

### Question 1

(i) From the figure (i) , if

$$DE \parallel BC$$

. Find

$$EC$$

.

(ii) From the figure (ii) , if

$$DE \parallel BC$$

.

$$AD$$

in (ii).

**Solution**

Ans: Let us assume that

$$EC = x \text{ cm}$$

Given that  $DE \parallel BC$

But from basic proportionality theorem, we know that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore$$

$$EC = 2 \text{ cm}$$

Ans:

Let us assume that

$$AD = x \text{ cm}$$

Given that

$$DE \parallel BC.$$

But from basic proportionality theorem we know that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4$$

cm

## Question 2

(i) In a  $\triangle PQR$ ,

E

and

F

are any two points on the sides

PQ

and

PR

respectively. State whether

$EF \parallel QR$

for

$$PE = 3.9 \text{ cm}, EQ = 3 \text{ cm}, PF = 3.6 \text{ cm}$$

and

$$FR = 2.4 \text{ cm}$$

(iii) In a  $\triangle PQR$ ,

E

and

F

are any two points on the sides

PQ

and

PR

respectively. State whether

$$EF \parallel QR$$

for

$$PQ = 1.28 \text{ cm}, PR = 2.56 \text{ cm}, PE = 0.18 \text{ cm}$$

and

$$PF = 0.63 \text{ cm}$$

### Solution

Ans:

Given,

$$PE = 3.9 \text{ cm}, EQ = 3 \text{ cm}, PF = 3.6 \text{ cm}$$

,

$$FR = 2.4 \text{ cm}$$

$$\frac{PF}{EQ} = \frac{3.9}{3}$$

$$= 1.3$$

$$\frac{PF}{FR}$$

$$=$$

$$\frac{3.6}{2.4}$$

$$= 1.5$$

Hence,  $\frac{PE}{EQ}$

$$\neq$$

$$\frac{PF}{FR}$$

Therefore ,

EF

is parallel to

QR

.

(ii) In a  $\triangle PQR$ ,

E

and

F

are any two points on the sides

PQ

and

PR

respectively. State whether

$EF \parallel QR$

for

$$PE = 4 \text{ cm}, QE = 4.5 \text{ cm}, PF = 8 \text{ cm}$$

and

$$RF = 9 \text{ cm}$$

Ans:

$$PE = 4 \text{ cm}, QE = 4.5 \text{ cm}, PF = 8 \text{ cm}, RF = 9 \text{ cm}$$

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

$$\text{Hence, } \frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore,

EF

is parallel to

QR

.

Ans:

$$PQ = 1.28 \text{ cm}, PR = 2.56 \text{ cm}, PE = 0.18 \text{ cm}, PF = 0.36 \text{ cm}$$

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

$$\text{Hence, } \frac{PE}{PQ} = \frac{PF}{PR}$$

Therefore,

EF

is parallel to

QR

.

### Question 3

In the figure given below, if sides

$$LM \parallel CB$$

and

$$LN \parallel CD,$$

Show that  $\frac{AM}{AB} = \frac{AN}{AD}$

### Solution

Ans:

Given that in the figure,

$$LM \parallel CB$$

But from basic proportionality theorem, we know that

$$\frac{AM}{AB} = \frac{AL}{AC} \text{---(i)}$$

Also,

$$LN \parallel CD$$

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \text{---(ii)}$$

From (i) and (ii), we get

$$\frac{AM}{AB} = \frac{AN}{AD}$$

### Question 4

In the figure given below, if sides  $DE \parallel AC$  and  $DF \parallel AE$ . Show that  $\frac{BF}{FE} = \frac{BE}{EC}$

**Solution**

Ans:

In

$\triangle ABC, DE \parallel AC$

$$\therefore \frac{BD}{DA} = \frac{BE}{EC}$$

(By Basic proportionality theorem)

In

$\triangle BAE, DF \parallel AE$

$$\therefore \frac{BD}{DA} = \frac{BE}{FE}$$

By Basic proportionality theorem

From (i) and (ii), we get

$$\frac{BE}{EC} = \frac{BF}{FE}$$

**Question 5**

In the figure given below, if sides  $DE \parallel OQ$  and  $DF \parallel OR$ , Show that  $EF \parallel QR$

**Solution**

Ans:

In

$\triangle POQ, DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad \dots\dots\dots (i) \text{ By basic proportionality theorem}$$

$\Delta POR, DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO}$$

$\dots\dots\dots (ii) \text{ By basic proportionality theorem}$

From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore EF \parallel QR$  Converse of Basic proportionality theorem

### Question 6

In the figure given below,

A, B and C

are points on

OP, OQ and OR

respectively such that

$AB \parallel PQ$

and

$AC \parallel PR$

. Prove that

$BC \parallel QR$

.

### Solution

Ans:

In

$\triangle POQ, AB \parallel PQ$

$$\therefore \frac{OA}{OP} = \frac{OB}{PQ}$$

.....(i) By basic proportionality theorem

In

$\triangle POR, AC \parallel PR$

$$\therefore \frac{OA}{OP} = \frac{OC}{CR}$$

.....(ii) By basic proportionality theorem

From (i) and (ii), we get

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

$$\therefore BQ \parallel CR$$

Converse of Basic proportionality theorem

### Question 7

By using Basic proportionality theorem, Show that a line passing through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

### Solution

Ans:

Let us assume in the given figure in which

PQ

is a line segment passing through the mid-point

P

of line

AB

, such that

$PQ \parallel BC$

.

From basic proportionality theorem, we know that

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = 1$$

As

P

is the midpoint of

AB

,

$$\begin{aligned} AP &= PB \\ \Rightarrow AQ &= QC \end{aligned}$$

Or

Q

is the midpoint of

AC

### Question 8

By using Converse of basic proportionality theorem, Show that the line joined by the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

### Solution

Ans:

Let us assume that the given figure in which

PQ

is a line segment joined by the mid-points

P and Q

of lines

AB and AC

respectively.

i.e.,

$$AP = PB \text{ and } AQ = QC$$

Also it is clear that

$$\frac{AP}{PB} = 1 \text{ and}$$

$$\frac{AQ}{QC} = 1$$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, using basic proportionality theorem, we get

$PQ \parallel BC$

### Question 9

If

$ABCD$

is a trapezium where

$AB \parallel DC$

and its diagonals intersect each other at the point

$O$

. Prove that  $\frac{AO}{BO} = \frac{CO}{DO}$

### Solution

Ans:

Draw a line

$EF$

through point

$O$

, such that

In

$\triangle ADC$

,

$EO \parallel CD$

Using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{AO}{OC} \text{-----(i)}$$

In

$$\triangle ABD$$

$$, OE \parallel AB$$

So, using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{BO}{DO}$$

\_\_\_\_\_ (ii)

From equation (i) and (ii), we get

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\therefore \frac{AO}{BO} = \frac{CO}{DO}$$

### Question 10

The diagonals of a quadrilateral

ABCD

intersect each other at the point

O

such that  $\frac{AO}{BO} = \frac{CO}{DO}$  Prove that

ABCD

is a trapezium.

### Solution

Ans:

Let us assume the following figure for the given question.

Draw a line

$OE \parallel AB$

In

;ABD, OE||AB

Using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{BO}{OD} \text{---(i)}$$

However, it is given that

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\therefore \frac{AO}{CO} = \frac{BO}{DO} \text{----- (ii)}$$

From equations (i) and (ii), we get

$$\frac{AE}{ED} = \frac{AO}{OC}$$

$$\Rightarrow EO||DC$$

By the converse of basic proportionality theorem

$$\Rightarrow AB || OE || DC$$

$$\Rightarrow AB||CD$$

$\therefore$  ABCD

is a trapezium.

## Exercise 6.3

### Question 1

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

**Solution**

Ans:

1. In  $\triangle ABC$  and  $\triangle PQR$

$$\angle A = \angle P$$

$$\angle B = \angle Q$$

$$\angle C = \angle R$$

$\therefore$  By AAA criterion of similarity,  $\triangle ABC \sim \triangle PQR$

1. In  $\triangle ABC$  and  $\triangle QRP$

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{AC}{QP} = \frac{1}{2}$$

$\therefore$  By SSS criterion of similarity,  $\triangle ABC \sim \triangle QRP$

1. In  $\triangle LMP$  and  $\triangle DEF$

$$\frac{LM}{DE} = \frac{2.7}{4}, \frac{LP}{DF} = \frac{1}{2}$$

The sides are not in the equal ratios, Hence the two triangles are not similar.

1. In  $\triangle MNL$  and  $\triangle QPR$

$$\angle M = \angle Q$$

$$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$$

$\therefore$  By SAS criterion of similarity,  $\triangle MNL \sim \triangle QPR$

1. In  $\triangle ABC$  and  $\triangle EFD$

$$\angle A = \angle F,$$

$$\frac{AB}{FD} = \frac{BC}{ED} = \frac{1}{2}$$

$\therefore$  By SAS criterion of similarity,  $\triangle ABC \sim \triangle EFD$

1. In  $\triangle DEF$  and  $\triangle PQR$

Since, sum of angles of a triangle is  $180^\circ$ , Hence,  $\angle F = 30^\circ$  and  $\angle P = 70^\circ$

$$\angle D = \angle P$$

$$\angle E = \angle Q$$

$$\angle F = \angle R$$

$\therefore$  By AAA criterion of similarity,  $\triangle DEF \sim \triangle PQR$

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### Question 2

In the following figure,  $\triangle ODC \sim \triangle OBA$ ,  $\angle BOC = 125^\circ$  and  $\angle CDO = 70^\circ$ . Find  $\angle DOC$ ,  $\angle DCO$  and  $\angle OAB$

### Solution

Ans: Given:

$$\triangle ODC \sim \triangle OBA$$

$$\angle BOC = 125^\circ$$

$$\angle CDO = 70^\circ$$

To find:  $\angle DOC$ ,  $\angle DCO$  and  $\angle OAB$

Sol: Here,  $BD$  is a line,

So, we can apply a linear pair on it.

$$\angle BOC + \angle DOC = 180^\circ (\text{Linear Pair})$$

$$125^\circ + \angle DOC = 180^\circ$$

$$\angle DOC = 180^\circ - 125^\circ$$

$$\angle DOC = 180^\circ - 125^\circ$$

$$\angle DOC = 55^\circ$$

Now in  $\triangle DCO$

$$\angle CDO + \angle DCO + \angle DOC = 180^\circ$$

$$70^\circ + \angle DCO + 55^\circ = 180^\circ$$

$$125^\circ + \angle DCO = 180^\circ$$

$$\angle DCO = 180^\circ - 125^\circ$$

$$\angle DCO = 55^\circ$$

Now it is given that

$$\triangle ODC \sim \triangle OBA$$

Hence,

$$\angle DCO = \angle OAB$$

$$55^\circ = \angle OAB$$

$$\angle OAB = 55^\circ$$

Now in  $\triangle DCO$

$$\angle CDO + \angle DCO + \angle DOC = 180^\circ \quad (\text{Sum of all angles of triangle is } 180^\circ)$$

$$70^\circ + \angle DCO + 55^\circ = 180^\circ$$

$$125^\circ + \angle DCO = 180^\circ$$

$$\angle DCO = 180^\circ - 125^\circ \quad \angle DCO = 55^\circ$$

Now, it is given that

$$\triangle ODC \sim \triangle OBA$$

Hence.

$$\angle DCO = \angle OAB \quad (\text{Corresponding angles of a similar triangles are equal})$$

$$55^\circ = \angle OAB$$

**Question 3**

Diagonals AC and BD of a trapezium ABCD with  $AB \parallel DC$  intersect each other at the point O. Using a similarity criterion for two triangles, show that  $\frac{AO}{OC} = \frac{OB}{OD}$

**Solution**

Ans: In  $\triangle DOC$  and  $\triangle BOA$

$$\angle CDO = \angle ABO \text{ (Alternate interior angles as } AB \parallel CD)$$

$$\angle DCO = \angle BAO \text{ (Alternate interior angles as } AB \parallel CD)$$

$$\angle DOC = \angle BOA \text{ (Vertically opposite angles )}$$

$$\therefore \triangle DOC \sim \triangle BOA \text{ (AAA similarity criterion)}$$

$$\therefore \frac{DO}{BO} = \frac{OC}{OA} \text{ ( Corresponding sides are proportional)}$$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

**Question 4**

In the figure,  $\frac{QR}{QS} = \frac{QT}{PR}$  and  $\angle 1 = \angle 2$ . Show that  $\triangle PQS \sim \triangle TQR$

**Solution**

Ans: In  $\triangle PQR$ ,  $\angle PQR = \angle PRQ$

$$\therefore PQ = PR \text{ (i)}$$

Given

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP}$$

In  $\triangle PQS$  and  $\triangle TQR$ ,

$$\frac{QR}{QS} = \frac{QT}{QP} \text{ [ Using (ii) ]}$$

$$\angle Q = \angle Q$$

$\therefore \triangle PQS \sim \triangle TQR$  [ SAS similarity criterion ]

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### Question 5

S and T are point on sides PR and QR of  $\triangle PQR$  such that  $\angle P = \angle RTS$ . Show that  $\triangle RPQ \sim \triangle RTS$ .

### Solution

Ans: Given:  $\triangle PQR$

and the points  $S$  and  $T$  on sides PR and QR.

Such that  $\angle P = \angle RTS$

To Prove:  $\triangle RPQ \sim \triangle RTS$ .

Proof:

In  $\triangle RPQ$  and  $\triangle RTS$ .

$$\angle P = \angle RTS$$

(Given)

$$\text{And } \angle PRQ = \angle TRS = \angle R$$

(Common)

So,  $\triangle RPQ \sim \triangle RTS$ .

(AA similarity)

Hence proved

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### Question 6

In the following figure, if  $\triangle ABE \cong \triangle ACD$ , show that  $\triangle ADE \sim \triangle ABC$ .

### Solution

Ans:

We know that the corresponding portions of two triangles that are congruent to each other are equal.

The two triangles are comparable if one of their angles is equal to one of the other triangle's angles, and the sides that include these angles are proportionate.

For two triangles, this is known as the SAS (Side - Angle - Side) similarity criteria.

In  $\triangle ABE$  and  $\triangle ACD$

$$AD = AE (\triangle ABE \cong \triangle ACD \text{ given}) \dots\dots\dots (1)$$

$$AB = AC (\triangle ABE \cong \triangle ACD \text{ given})$$

Now Consider  $\triangle ADE$  and  $\triangle ABC$

$$\text{and } \angle DAE = \angle BAC \text{ (Common angle)}$$

Thus,  $\triangle ADE \sim \triangle ABC$  (SAS criterion)

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### Question 7

In the following figure, altitudes AD and CE of  $\triangle ABC$  intersect each other at the point, P. Show, that:

1.  $\triangle AEP \sim \triangle CDP$

## Solution

Ans: When two angles from one triangle are equivalent to two angles from another triangle, the two triangles are said to be comparable.

For two triangles, this is known as the AA similarity criteria.

In  $\triangle AEP$  and  $\triangle CDP$

[  $\because CE \perp AB$  and  $AD \perp BC$ ; altitudes ]

$\angle APE = \angle CPD$  (Vertically opposite angles)

$\Rightarrow \triangle AEP \sim \triangle CPD$  (AA criterion)

1.  $\triangle ABD \sim \triangle CBE$

Ans: When two angles from one triangle are equivalent to two angles from another triangle, the two triangles are said to be comparable.

For two triangles, this is known as the AA similarity criteria.

In  $\triangle ABD$  and  $\triangle CBE$

$\angle ADB = \angle CEB = 90^\circ$

$\angle ABD = \angle CBE$  (Common angle)

$\Rightarrow \triangle ABD \sim \triangle CBE$  (AA criterion)

1.  $\triangle AEP \sim \triangle ADB$

Ans: When two angles from one triangle are equivalent to two angles from another triangle, the two triangles are said to be comparable.

For two triangles, this is known as the AA similarity criteria.

In  $\triangle AEP$  and  $\triangle ADB$

$\angle AEP = \angle ADB = 90^\circ$

$\angle PAE = \angle BAD$  (Common angle)

$$\Rightarrow \triangle AEP \sim \triangle ADB \text{ (AA criterion)}$$

$$1. \triangle PDC \sim \triangle BEC$$

Ans: When two angles from one triangle are equivalent to two angles from another triangle, the two triangles are said to be comparable.

For two triangles, this is known as the AA similarity criteria.

In  $\triangle PDC$  and  $\triangle BEC$

$$\angle PDC = \angle BEC = 90^\circ$$

$$\angle PCD = \angle BCE \text{ (Common angle)}$$

$$\Rightarrow \triangle PDC \sim \triangle BEC \text{ (AA criterion)}$$

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### Question 8

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that  $\triangle ABE \sim \triangle CFB$

### Solution

Ans:

In  $\triangle ABE$  and  $\triangle CFB$ ,

$$\angle A = \angle C \text{ (Opposite angles of a parallelogram)}$$

$$\angle AEB = \angle CBF \text{ (Alternate interior angles as } AE \parallel BC)$$

$$\therefore \triangle ABE \sim \triangle CFB \text{ (By AA similarity criterion)}$$

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**Question 9**

In the following figure,  $\triangle ABC$  and  $\triangle AMP$  are two right triangles, right angled at B and M respectively, prove that:

1.  $\triangle ABC \sim \triangle AMP$

2.  $\frac{CA}{PA} = \frac{BC}{MP}$

**Solution**

Ans: In  $\triangle ABC$  and  $\triangle AMP$

$$\angle ABC = \angle AMP \text{ (Each } 90^\circ) \quad \angle A = \angle A \text{ (Common)}$$

$$\therefore \triangle ABC \sim \triangle AMP \text{ (By AA similarity criterion)}$$

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \text{ (Corresponding sides of similar triangles are proportional)}$$

**Question 10**

CD and GH are respectively the bisectors of  $\angle ACB$  and  $\angle EGF$  such that D and H lie on sides AB and FE of  $\triangle ABC$  and  $\triangle EFG$  respectively. If  $\triangle ABC \sim \triangle FEG$ , Show that:

(i)  $\frac{CD}{GH} = \frac{AC}{FG}$

(ii)  $\triangle DCB \sim \triangle HGE$

(iii)  $\triangle DCA \sim \triangle HGF$

**Solution**

Ans: It is given that  $\triangle ABC \sim \triangle FEG$ .

$$\therefore \angle A = \angle F, \angle B = \angle E, \text{ and } \angle ACB = \angle FGE$$

$$\text{Since, } \angle ACB = \angle FGE$$

$\therefore \angle ACD = \angle FGH$  (Angle bisector)

And,  $\angle DCB = \angle HGE$  (Angle bisector)

In  $\triangle ACD$  and  $\triangle FGH$ ,

$\angle A = \angle F$  (Proved above)

$\angle ACD = \angle FGH$  (Proved above)

$\therefore \triangle ACD \sim \triangle FGH$  (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

Ans: In  $\triangle DCB$  and  $\triangle HGE$ ,

$\angle DCB = \angle HGE$  (Proved above)

$\angle B = \angle E$  (Proved above)

$\therefore \triangle DCB \sim \triangle HGE$  (By AA similarity criterion)

Ans: In  $\triangle DCA$  and  $\triangle HGF$ ,

$\angle ACD = \angle FGH$  (Proved above)

$\angle A = \angle F$  (Proved above)

$\therefore \triangle DCA \sim \triangle HGF$  (By AA similarity criterion)

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### Question 11

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with  $AB = AC$ . If  $AD \perp BC$  and  $EF \perp AC$ , prove that  $\triangle ABD \sim \triangle ECF$

**Solution**

Ans: It is given that  $\triangle ABC$  is an isosceles triangle.

$$\therefore AB = AC$$

$$\Rightarrow \angle ABD = \angle ECF$$

In  $\triangle ABD$  and  $\triangle ECF$

$$\angle ADB = \angle EFC \text{ (Each } 90^\circ \text{)}$$

$$\angle BAD = \angle CEF \text{ (Proved above)}$$

$$\therefore \triangle ABD \sim \triangle ECF \text{ (By using AA similarity criterion)}$$

**Question 12**

Sides  $AB$  and  $BC$  and median  $AD$  of a triangle  $ABC$  are respectively proportional to sides  $PQ$  and  $QR$  and median  $PM$  of  $\triangle PQR$  (see the given figure). Show that  $\triangle ABC \sim \triangle PQR$ .

**Solution**

Ans: Median equally divides the opposite side.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2}$$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In  $\triangle ABD$  and  $\triangle PQM$ ,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \text{ (Proved above)}$$

$$\therefore \triangle ABD \sim \triangle PQM \text{ (By SSS similarity criterion)}$$

$\Rightarrow \angle ABD = \angle PQM$  (Corresponding angles of similar triangles)

In  $\triangle ABC$  and  $\triangle PQR$ ,

$\angle ABD = \angle PQM$  (Proved above)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$  (By SAS similarity criterion)

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### Question 13

D is a point on the side BC of a triangle ABC such that  $\angle ADC = \angle BAC$ . Show that  $CA^2 = CB \cdot CD$

### Solution

Ans: In  $\triangle ADC$  and  $\triangle BAC$ ,

$\angle ADC = \angle BAC$  (Given)  $\angle ACD = \angle BCA$  (Common angle)

$\therefore \triangle ADC \sim \triangle BAC$  (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.  $\therefore \frac{CA}{CB} = \frac{CD}{CA}$

$$\Rightarrow CA^2 = CB \cdot CD$$

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### Question 14

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that  $\triangle ABC \sim \triangle PQR$

### Solution

Ans: Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point  $E$  and  $L$  respectively, such that  $AD = DE$  and  $PM = ML$ .

Then, join  $B$  to  $E$ ,  $C$  to  $E$ ,  $Q$  to  $L$ , and  $R$  to  $L$ .

We know that medians divide opposite sides.

Therefore,  $BD = DC$  and  $QM = MR$

Also,  $AD = DE$  (By construction)

And,  $PM = ML$  (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D. Therefore, quadrilateral ABEC is a parallelogram.

$\therefore AC = BE$  and  $AB = EC$  (Opposite sides of a parallelogram are equal) Similarly, we can prove that quadrilateral PQLR is a parallelogram and  $PR = QL$ ,  $PQ = LR$

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

$\therefore \triangle ABE \sim \triangle PQL$  (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

$$\therefore \angle BAE = \angle QPL \dots (1)$$

Similarly, it can be proved that  $\triangle AEC \sim \triangle PLR$  and

$$\angle CAE = \angle RPL \dots (2)$$

Adding equation (1) and (2), we obtain

$$\angle BAE + \angle CAE = \angle QPL + \angle RPL$$

$$\Rightarrow \angle CAB = \angle RPQ \dots (3)$$

In  $\triangle ABC$  and  $\triangle PQR$ ,

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ (Given)}$$

$$\angle CAB = \angle RPQ \text{ [Using equation (3)]}$$

$\therefore \triangle ABC \sim \triangle PQR$  (By SAS similarity criterion)

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### Question 15

A vertical pole of a length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

### Solution

Ans: Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore,  $\angle DCF = \angle BAE$

And,  $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$  (Tower and pole are vertical to the ground)

$\therefore \triangle ABE \sim \triangle CDF$  (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

Therefore, the height of the tower will be 42 metres.

**Question 16**

If AD and PM are medians of triangles ABC and PQR, respectively where  $\triangle ABC \sim \triangle PQR$  Prove that  $\frac{AB}{PQ} = \frac{AD}{PM}$

**Solution**

Ans: It is given that  $\triangle ABC \sim \triangle PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots (1)$$

$$\text{Also, } \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$$

Since AD and PM are medians, they will divide their opposite sides.  $\therefore BD = \frac{BC}{2}$  and  $QM = \frac{QR}{2}$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM}$$

In  $\triangle ABD$  and  $\triangle PQM$ ,

$$\angle B = \angle Q \text{ (Using equation (2))}$$

$$\frac{AB}{PQ} = \frac{BD}{QM}$$

(Using equation (4))

$\therefore \triangle ABD \sim \triangle PQM$  (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$