

Chapter 4: Quadratic Equations

NCERT · Class 10 · Maths

Exercise 4.1

Question 1

Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii)

$$x^3 - 4x^2 - x + 1 = (x - 2)^3$$

Solution

Ans: We are given an equation: $(x + 1)^2 = 2(x - 3)$

We will simplify the given equation.

$$(x + 1)^2 = 2(x - 3)$$

$$\Rightarrow x^2 + 2x + 1 = 2x - 6$$

$$\Rightarrow x^2 + 1 = -6$$

$$\Rightarrow x^2 + 7 = 0$$

The equation obtained after simplifying is of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

Ans: We are given an equation: $x^2 - 2x = (-2)(3 - x)$

We will simplify the given equation.

$$x^2 - 2x = (-2)(3 - x)$$

$$\Rightarrow x^2 - 2x = -6 + 2x$$

$$\Rightarrow x^2 - 4x + 6 = 0$$

The equation obtained after simplifying is of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

Ans: We are given an equation: $(x - 2)(x + 1) = (x - 1)(x + 3)$

We will simplify the given equation.

$$(x - 2)(x + 1) = (x - 1)(x + 3)$$

$$\Rightarrow x^2 - 2x + x - 2 = x^2 - x + 3x - 3$$

$$\Rightarrow -2x + x - 2 = -x + 3x - 3$$

$$\Rightarrow -x - 2 = 2x - 3$$

$$\Rightarrow 3x - 1 = 0$$

This equation obtained is not of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

Ans: We are given an equation: $(x - 3)(2x + 1) = x(x + 5)$

We will simplify the given equation.

$$(x - 3)(2x + 1) = x(x + 5)$$

$$\Rightarrow 2x^2 - 6x + x - 3 = x^2 + 5x$$

$$\Rightarrow x^2 - 10x - 3 = 0$$

The equation obtained after simplifying is of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

Ans: We are given an equation: $(2x - 1)(x - 3) = (x + 5)(x - 1)$

We will simplify the given equation.

$$\Rightarrow 2x^2 - x - 6x + 3 = x^2 + 5x - x - 5$$

$$\Rightarrow x^2 - 7x + 3 = 4x - 5$$

$$\Rightarrow x^2 - 11x + 8 = 0$$

The equation obtained after simplifying is of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

Ans: We are given an equation: $x^2 + 3x + 1 = (x - 2)^2$

We will simplify the given equation.

$$x^2 + 3x + 1 = (x - 2)^2$$

$$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$$

$$\Rightarrow 3x + 1 = -4x + 4$$

$$\Rightarrow 7x - 3 = 0$$

This equation obtained is not of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

Ans: We are given an equation: $(x + 2)^3 = 2x(x^2 - 1)$

We will simplify the given equation.

$$(x + 2)^3 = 2x(x^2 - 1)$$

$$\Rightarrow x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$$

$$\Rightarrow 6x^2 + 14x + 8 = x^3$$

$$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$$

This equation obtained is not of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

Ans: We are given an equation: $x^3 - 4x^2 - x + 1 = (x - 2)^3$

We will simplify the given equation.

$$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$$

$$\Rightarrow -4x^2 - x + 1 = -6x^2 + 12x - 8$$

$$\Rightarrow 2x^2 - 13x + 9 = 0$$

This equation obtained is of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

Question 2

Represent the following situations in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m^2 . The length of the plot (in meters) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Solution

Ans: It is given that the area of rectangular plot is 528 m^2 .

Let us assume the breadth of the plot be $b = x \text{ m}$.

It is given that the length is one more than twice the breadth.

So, the length will be $l = (2x + 1) \text{ m}$.

The area of the rectangle will be $A = l \times b$

$$\therefore l \times b = 528$$

$$\Rightarrow x(2x + 1) = 528$$

$$\Rightarrow 2x^2 + x - 528 = 0$$

Therefore, the required quadratic equation is $2x^2 + x - 528 = 0$.

Ans: It is given that the product of two consecutive positive integers is 306.

If we assume the first integer to be x , then the next consecutive integer will be $(x+1)$.

$$\text{So, we can write } x(x + 1) = 306$$

$$\Rightarrow x^2 + x - 306 = 0$$

Therefore, the required quadratic equation is $x^2 + x - 306 = 0$.

Ans: Let us assume Rohan's age be x .

His mother is 26 years older than him.

$$\text{So, his mother's age will be } (x + 26)$$

$$\text{After 3 years, Rohan's age} = x + 3$$

$$\text{His mother's age} = x + 26 + 3 = x + 29$$

It is given that after 3 years, the product of their ages is 360.

$$\therefore (x + 3)(x + 29) = 360$$

$$\Rightarrow x^2 + 32x + 87 = 360$$

$$\Rightarrow x^2 + 32x - 273 = 0$$

Therefore, the required quadratic equation is $x^2 + 32x - 273 = 0$.

Ans: Let us assume the speed of the train be x km/h.

The train travels a distance of 480 km at a uniform speed.

So, the time taken to travel the given distance = $\frac{480}{x}$ hrs.

It is given that the train would take 3 hours more if the speed had been 8 km/h less.

Now, the new speed of the train = $(x - 8)$ km/h.

The new time taken to travel the same distance = $\left(\frac{480}{x} + 3\right)$ hr.

We know that speed $\times$ time = distance

$$\therefore (x - 8) \left(\frac{480}{x} + 3\right) = 480$$

$$\Rightarrow 480 + 3x - \frac{3840}{x} - 24 = 480$$

$$\Rightarrow 3x - \frac{3840}{x} = 24$$

$$\Rightarrow 3x^2 - 24x - 3840 = 0$$

$$\Rightarrow x^2 - 8x - 1280 = 0$$

Therefore, the required quadratic equation is $x^2 - 8x - 1280 = 0$.

Exercise 4.2

Question 1

Find the roots of the following quadratic equations by factorisation:

i. $x^2 - 3x - 10 = 0$

ii. $2x^2 + x - 6 = 0$

iii. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

iv. $2x^2 - x + \frac{1}{8} = 0$

v. $100x^2 - 20x + 1 = 0$

Solution

Ans: $x^2 - 3x - 10 = 0$

$$\Rightarrow x^2 - 5x + 2x - 10$$

$$\Rightarrow x(x-5) + 2(x-5)$$

$$\Rightarrow (x-5)(x+2)$$

Therefore, roots of this equation are –

$$x-5=0 \text{ or } x+2=0$$

$$\text{i.e } x=5 \text{ or } x=-2$$

$$\text{Ans: } 2x^2 + x - 6 = 0$$

$$\Rightarrow 2x^2 + 4x - 3x - 6$$

$$\Rightarrow 2x(x+2) - 3(x+2)$$

$$\Rightarrow (x+2)(2x-3)$$

Therefore, roots of this equation are –

$$x+2=0 \text{ or } 2x-3=0$$

$$\text{i.e } x=-2 \text{ or } x=\frac{3}{2}$$

$$\text{Ans: } \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2}$$

$$\Rightarrow x(\sqrt{2}x+5) + \sqrt{2}(\sqrt{2}x+5)$$

$$\Rightarrow (\sqrt{2}x+5)(x+\sqrt{2})$$

Therefore, roots of this equation are –

$$\sqrt{2}x+5=0 \text{ or } x+\sqrt{2}=0$$

$$\text{i.e } x = \frac{-5}{\sqrt{2}} \text{ or } x = -\sqrt{2}$$

$$\text{Ans: } 2x^2 - x + \frac{1}{8} = 0$$

$$\Rightarrow \frac{1}{8} (16x^2 - 8x + 1)$$

$$\Rightarrow \frac{1}{8} (4x(4x - 1) - 1(4x - 1))$$

$$\Rightarrow (4x - 1)^2$$

Therefore, roots of this equation are –

$$4x - 1 = 0 \text{ or } 4x - 1 = 0$$

$$\text{i.e. } x = \frac{1}{4} \text{ or } x = \frac{1}{4}$$

$$\text{Ans: } 100x^2 - 20x + 1 = 0$$

$$\Rightarrow 100x^2 - 10x - 10x + 1$$

$$\Rightarrow 10x(10x - 1) - 1(10x - 1)$$

$$\Rightarrow (10x - 1)(10x - 1)$$

Therefore, roots of this equation are –

$$(10x - 1) = 0$$

or

$$(10x - 1) = 0$$

$$\text{i.e. } x = \frac{1}{10} \text{ or } x = \frac{1}{10}$$

Question 2

i. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

ii. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total

cost of production was Rs 750. Find out the number of toys produced on that day.

Solution

Ans: Let the number of John's marbles be x .

Thus, the number of Jivanti's marbles is $45-x$.

According to question i.e.,

After losing 5 marbles.

Number of John's marbles be $x-5$

And the number of Jivanti's marble is $40-x$.

Therefore, $(x-5)(40-x)=124$

$$\Rightarrow x^2 - 45x + 324 = 0$$

$$\Rightarrow x^2 - 36x - 9x + 324 = 0$$

$$\Rightarrow x(x-36) - 9(x-36) = 0$$

$$\Rightarrow (x-36)(x-9) = 0$$

So now,

Case 1: If $x-36=0$ i.e $x=36$

So, the number of John's marbles is 36.

Thus, the number of Jivanti's marbles is 9.

Case 2: If $x-9=0$ i.e $x=9$

So, the number of John's marbles will be 9.

Thus, the number of Jivanti's marbles is 36.

Ans: Let the number of toys produced be x .

Therefore, Cost of production of each toy is Rs $(55-x)$.

Thus, $(55-x)x=750$

$$\Rightarrow x^2 - 55x + 750 = 0$$

$$\Rightarrow x^2 - 25x - 30x + 750 = 0$$

$$\Rightarrow x(x - 25) - 30(x - 25) = 0$$

$$\Rightarrow (x - 25)(x - 30) = 0$$

Case 1: If $x - 25 = 0$ i.e $x = 25$

So, the number of toys will be 25.

Case 2: If $x - 30 = 0$ i.e $x = 30$

So, the number of toys will be 30.

Question 3

Find two numbers whose sum is 27 and product is 182.

Solution

Ans: Let the first number be x ,

Thus, the second number is $27 - x$.

Therefore,

$$x(27 - x) = 182$$

$$\Rightarrow x^2 - 27x + 182 = 0$$

$$\Rightarrow x^2 - 13x - 14x + 182 = 0$$

$$\Rightarrow x(x - 13) - 14(x - 13) = 0$$

$$\Rightarrow (x - 13)(x - 14) = 0$$

Case 1: If $x - 13 = 0$ i.e $x = 13$

So, the first number be 13 ,

Thus, the second number is 14.

Case 2: If $x-14=0$ i.e $x=14$

So, the first number is 14.

Thus, the second number is 13.

Question 4

Find two consecutive positive integers, sum of whose squares is 365.

Solution

Ans: Let the consecutive positive integers be x and $x+1$.

$$\text{Thus, } x^2 + (x+1)^2 = 365$$

$$\Rightarrow x^2 + x^2 + 1 + 2x = 365$$

$$\Rightarrow 2x^2 + 2x - 364 = 0$$

$$\Rightarrow x^2 + x - 182 = 0$$

$$\Rightarrow x^2 + 14x - 13x - 182 = 0$$

$$\Rightarrow x(x+14) - 13(x+14) = 0$$

$$\Rightarrow (x+14)(x-13) = 0$$

Case 1: If $x+14=0$ i.e $x=-14$.

This case is rejected because the number is positive.

Case 2: If $x-13=0$ i.e $x=13$

So, the first number is 13.

Thus, the second number is 14.

Hence, the two consecutive positive integers are 13 and 14.

Question 5

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution

Ans: Let the base of the right-angled triangle be x cm.

Its altitude is $(x-7)$ cm.

Thus, by pythagoras theorem-

$$\text{base}^2 + \text{altitude}^2 = \text{hypotenuse}^2$$

$$\therefore x^2 + (x-7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 + 12x + 5x - 60 = 0$$

$$\Rightarrow x(x-12) + 5(x-12) = 0$$

$$\Rightarrow (x-12)(x+5) = 0$$

Case 1: If $x-12=0$ i.e $x=12$.

So, the base of the right-angled triangle be 12 cm and Its altitude be 5cm

Case 2: If $x+5=0$ i.e $x=-5$

This case is rejected because the side is always positive.

Hence, the base of the right-angled triangle is 12 cm and its altitude is 5cm.

Question 6

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Solution

Ans: Let the number of articles produced be x .

Therefore, the cost of production of each article is Rs $(2x+3)$.

$$\text{Thus, } x(2x+3) = 90$$

$$\Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x+15) - 6(2x+15) = 0$$

$$\Rightarrow (2x+15)(x-6) = 0$$

$$\text{Case 1: If } 2x-15=0 \text{ i.e. } x = \frac{-15}{2}.$$

This case is rejected because the number of articles is always positive.

$$\text{Case 2: If } x-6=0 \text{ i.e. } x=6$$

Hence, the number of articles produced will be 6.

Therefore, the cost of production of each article is Rs15.

Exercise 4.3

Question 1

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them;

I. $2x^2 - 3x + 5 = 0$

II. $3x^2 - 4\sqrt{3}x + 4 = 0$

III. $2x^2 - 6x + 3 = 0$

Solution

Ans: Given: Quadratic equations

To find: nature of the roots.

We know that for a quadratic equation $ax^2 + bx + c = 0$

$$\text{Discriminant} = b^2 - 4ac$$

(A) If $b^2 - 4ac > 0 \rightarrow$ two distinct real roots

(B) If $b^2 - 4ac = 0 \rightarrow$ two equal real roots

(C) If $b^2 - 4ac < 0 \rightarrow$ no real roots

(I) $2x^2 - 3x + 5 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we obtain $a = 2$, $b = -3$, $c = 5$

$$\begin{aligned}\text{Discriminant : } b^2 - 4ac &= (-3)^2 - 4(2)(5) = 9 - 40 \\ &= -31\end{aligned}$$

$$\text{As } b^2 - 4ac < 0$$

Therefore, no real root is possible for the given equation.

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we obtain $a = 3$, $b = -4\sqrt{3}$, $c = 4$

$$\begin{aligned}\text{Discriminant : } b^2 - 4ac &= (-4\sqrt{3})^2 - 4(3)(4) \\ &= 48 - 48 = 0\end{aligned}$$

As $b^2 - 4ac = 0$

Therefore, real roots exist for the given equation and they are equal to each other. And the roots will be

$$\frac{-b}{2a} \text{ and } \frac{-b}{2a}.$$

$$\frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2 \times 3} = \frac{-4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$

(III) $2x^2 - 6x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we obtain $a = 2$, $b = -6$, $c = 3$

$$\text{Discriminant} = b^2 - 4ac = (-6)^2 - 4(2)(3)$$

$$= 36 - 24 = 12$$

As $b^2 - 4ac > 0$

Therefore, distinct real roots exist for the given equation as follows. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the root are $\frac{3 + \sqrt{3}}{2}$ or $\frac{3 - \sqrt{3}}{2}$

Question 2

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

I. $2x^2 + kx + 3 = 0$

$$\text{II. } kx(x - 2) + 6 = 0$$

Solution

Ans: Given: A quadratic equation

To find: k if the roots are equal

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its Discriminant $(b^2 - 4ac)$ will be 0 .

$$\text{(I) } 2x^2 + kx + 3 = 0$$

Comparing equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = k, c = 3$$

$$\begin{aligned}\text{Discriminant} &= b^2 - 4ac = (k)^2 - 4(2)(3) \\ &= k^2 - 24\end{aligned}$$

For equal roots, Discriminant = 0

$$k^2 - 24 = 0$$

$$k^2 = 24$$

$$k = \pm\sqrt{24} = \pm 2\sqrt{6}$$

$$\text{(II) } kx(x - 2) + 6 = 0$$

$$\text{or } kx^2 - 2kx + 6 = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$, we obtain $a = k, b = -2k, c = 6$

$$\begin{aligned}\text{Discriminant} &= b^2 - 4ac = (-2k)^2 - 4(k)(6) \\ &= 4k^2 - 24k\end{aligned}$$

For equal roots, $b^2 - 4ac = 0$

$$4k^2 - 24k = 0$$

$$4k(k - 6) = 0$$

Either $4k = 0$ or $k = 6 = 0$

$$k = 0 \text{ or } k = 6$$

However, if $k = 0$, then the equation will not have the terms ' x^2 ' and ' x '.

Therefore, if the given equation has two equal roots, k should be 6 only.

Question 3

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2 ? If so, find its length and breadth.

Solution

Ans: Given: A rectangular mango grove.

To find: its dimensions

Let us consider the breadth of mango grove be l .

Length of mango grove will be $2l$. Area of mango grove = $(2l)(l) = 2l^2 = 800$

$$l^2 = \frac{800}{2} = 400$$

$$l^2 - 400 = 0$$

Comparing the given equation with $al^2 + bl + c = 0$, we obtain $a = 1, b = 0, c = -400$

$$\text{Discriminant} = b^2 - 4ac = (0)^2 - 4 \times (1) \times (-400) = 1600$$

$$\text{Here, } b^2 - 4ac > 0$$

Therefore, the equation will have real roots.

$$l = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$l = \frac{-0 \pm \sqrt{1600}}{2 \times 1}$$

And Therefore, the desired rectangular mango grove can be designed. $l = \pm 20$

However, length cannot be negative. Therefore, breadth of mango grove = 20 m

$$\text{Length of mango grove} = 2 \times 20 = 40 \text{ m}$$

Question 4

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48 .

Solution

Ans: Given: A situation.

To find: Predict the situation whether it is possible or not?

Let us consider the age of one friend be x years.

Age of the other friend will be $(20 - x)$ years.

4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4) = (16 - x)$ years

Given that, $(x - 4)(16 - x) = 48$

$$16x - 64 - x^2 + 4x = 48$$

$$-x^2 + 20x - 112 = 0$$

$$x^2 - 20x + 112 = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$, we obtain $a = 1$, $b = -20$, $c = 112$

$$\text{Discriminant} = b^2 - 4ac = (-20)^2 - 4(1)(112)$$

$$= 400 - 448 = -48$$

$$\text{As } b^2 - 4ac < 0$$

Therefore, no real root is possible for the given equation and Therefore, the given situation is not possible.

Question 5

Is it possible to design a rectangular park of perimeter 80 and area 400 m^2 ? If so, find its length and breadth

Solution

Ans: Given: A situation.

To find: Predict the situation whether it is possible or not?

Let us consider the length and breadth of the park be l and b .

$$\text{Perimeter} = 2(l + b) = 80 \quad l + b = 40$$

$$\text{Or, } b = 40 - l$$

$$\text{Area} = l \times b = l(40 - l) = 40l - l^2$$

$$40l - l^2 = 400$$

$$l^2 - 40l + 400 = 0$$

Comparing the given equation with $al^2 + bl + c = 0$, we obtain

$$a = 1, \quad b = -40, \quad c = 400$$

$$\text{Discriminate} = b^2 - 4ac = (-40)^2 - 4(1)(400)$$

$$= 1600 - 1600 = 0$$

$$\text{As } b^2 - 4ac = 0$$

Therefore, the given equation has equal real roots. And Therefore, the given situation

is possible. Root of the given equation, $l = -\frac{b}{2a}$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m