

Chapter 2: Polynomials

NCERT · Class 10 · Maths

Exercise 2.1

Question 1

The graphs of

$$y=p(x)$$

are given in following figure, for some Polynomials

$$p(x)$$

. Find the number of zeroes of

$$p(x)$$

, in each case.

(i).

(ii).

(iii).

(iv).

(v).

(vi).

Solution

Ans: The graph does not intersect the

x-axis

at any point. Therefore, it does not have any zeroes.

Ans: The graph intersects at the

x-axis

at only

1

point. Therefore, the number of zeroes is

1

.

Ans: The graph intersects at the

x-axis

at

3

points. Therefore, the number of zeroes is

3

.

Ans: The graph intersects at the

x-axis

at

2

points. Therefore, the number of zeroes is

2

.

Ans: The graph intersects at the

x-axis

at

4

points. Therefore, the number of zeroes is

4

.

Ans: The graph intersects at the

x-axis

at

3

points. Therefore, the number of zeroes is

3

.

Exercise 2.2

Question 1

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i)

$$x^2 - 2x - 8$$

(ii)

$$4s^2 - 4s + 1$$

(iii)

$$6x^2 - 3 - 7x$$

(iv)

$$4u^2 + 8u$$

(v)

$$t^2 - 15$$

(vi)

$$3x^2 - x - 4$$

Solution

Ans:

Given:

$$x^2 - 2x - 8$$

.

Now factorize the given polynomial to get the roots.

$$\Rightarrow (x - 4)(x + 2)$$

The value of

$$x^2 - 2x - 8$$

is zero.

when

$$x - 4 = 0$$

or

$$x + 2 = 0$$

. i.e.,

$$x = 4$$

or

$$x = -2$$

Therefore, the zeroes of

$$x^2 - 2x - 8$$

are

$$4$$

and

$$-2$$

.

Now, Sum of zeroes

$$= 4 - 2 = 2 = -\frac{2}{1} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\therefore \text{Sum of zeroes} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes

$$= 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

Ans:

Given:

$$4s^2 - 4s + 1$$

Now factorize the given polynomial to get the roots.

$$\Rightarrow (2s - 1)^2$$

The value of

$$4s^2 - 4s + 1$$

is zero.

when

$$2s - 1 = 0$$

,

$$2s - 1 = 0$$

. i.e.,

$$s = \frac{1}{2}$$

and

$$s = \frac{1}{2}$$

Therefore, the zeroes of

$$4s^2 - 4s + 1$$

are

$$\frac{1}{2}$$

and

$$\frac{1}{2}$$

.

Now, Sum of zeroes

$$= \frac{1}{2} + \frac{1}{2} = 1 = \frac{(-4)}{4} = \frac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$$

$$\therefore \text{Sum of zeroes} = \frac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$$

Product of zeroes

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

$$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}.$$

Ans:

Given:

$$6x^2 - 3 - 7x$$

$$\Rightarrow 6x^2 - 7x - 3$$

Now factorize the given polynomial to get the roots.

$$\Rightarrow (3x+1)(2x-3)$$

The value of

$$6x^2 - 3 - 7x$$

is zero.

when

$$3x+1=0$$

or

$$2x-3=0$$

. i.e.,

$$x = \frac{-1}{3}$$

or

$$x = \frac{3}{2}$$

.

Therefore, the zeroes of

$$6x^2 - 3 - 7x$$

are

$$\frac{-1}{3}$$

and

$$\frac{3}{2}$$

.

Now, Sum of zeroes

$$= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\therefore \text{Sum of zeroes} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes

$$= \frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Ans:

Given:

$$4u^2 + 8u$$

$$\Rightarrow 4u^2 + 8u + 0$$

$$\Rightarrow 4u(u+2)$$

The value of

$$4u^2+8u$$

is zero.

when

$$4u=0$$

or

$$u+2=0$$

. i.e.,

$$u = 0$$

or

$$u = -2$$

Therefore, the zeroes of

$$4u^2+8u$$

are

$$0$$

and

$$-2$$

.

Now, Sum of zeroes

$$= 0 + (-2) = -2 = \frac{-8}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

$$\therefore \text{Sum of zeroes} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

Product of zeroes

$$= 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

$$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

Ans:

Given:

$$\begin{aligned} & t^2 - 15 \\ \Rightarrow & t^2 - 0t - 15 \end{aligned}$$

Now factorize the given polynomial to get the roots.

$$\Rightarrow (t - \sqrt{15})(t + \sqrt{15})$$

The value of

$$t^2 - 15$$

is zero.

when

$$t - \sqrt{15} = 0$$

or

$$t + \sqrt{15} = 0$$

, i.e.,

$$t = \sqrt{15}$$

or

$$t = -\sqrt{15}$$

Therefore, the zeroes of

$$t^2 - 15$$

are

$$\sqrt{15}$$

and

$$-\sqrt{15}$$

.

Now, Sum of zeroes

$$= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$$

$$\therefore \text{Sum of zeroes} = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$$

Product of zeroes

$$= (\sqrt{15}) \times (-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } t^2}$$

$$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } t^2}.$$

Ans:

Given:

$$3x^2 - x - 4$$

Now factorize the given polynomial to get the roots.

$$\Rightarrow (3x - 4)(x + 1)$$

The value of

$$3x^2 - x - 4$$

is zero.

when

$$3x - 4 = 0$$

or

$$x+1=0$$

, i.e.,

$$x = -\frac{4}{3}$$

or

$$x = -1$$

Therefore, the zeroes of

$$3x^2 - x - 4$$

are

$$\frac{4}{3}$$

and

$$-1$$

.

Now, Sum of zeroes

$$= \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\therefore \text{Sum of zeroes} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes

$$= \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

Question 2

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i)

$$\frac{1}{4}, -1$$

(ii)

$$\sqrt{2}, \frac{1}{3}$$

(iii)

$$0, \sqrt{5}$$

(iv)

$$1, 1$$

(v)

$$-\frac{1}{4}, \frac{1}{4}$$

(vi)

$$4, 1$$

Solution

Ans:

Given:

$$\frac{1}{4}, -1$$

Let the zeroes of polynomial be

$$\alpha$$

and

$$\beta$$

.

Then,

$$\alpha + \beta = \frac{1}{4}$$

$$\alpha\beta = -1$$

Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

.

$$\Rightarrow x^2 - \frac{1}{4}x - 1$$

$$\Rightarrow 4x^2 - x - 4$$

Therefore, the quadratic polynomial is

$$4x^2 - x - 4$$

.

Ans:

Given:

$$\sqrt{2}, \frac{1}{3}$$

Let the zeroes of polynomial be

$$\alpha$$

and

$$\beta$$

.

Then,

$$\alpha + \beta = \sqrt{2}$$

$$\alpha\beta = \frac{1}{3}$$

Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

.

$$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3}$$

$$\Rightarrow 3x^2 - 3\sqrt{2}x + 1$$

Therefore, the quadratic polynomial is

$$3x^2 - 3\sqrt{2}x + 1$$

.

Ans:

Given:

$$0, \sqrt{5}$$

Let the zeroes of polynomial be

$$\alpha$$

and

$$\beta$$

.

Then,

$$\alpha + \beta = 0$$

$$\alpha\beta = \sqrt{5}$$

Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

.

$$\Rightarrow x^2 - 0x + \sqrt{5}$$

$$\Rightarrow x^2 + \sqrt{5}$$

Therefore, the quadratic polynomial is

$$x^2 + \sqrt{5}$$

.

Ans:

Given:

$$1, 1$$

Let the zeroes of polynomial be

$$\alpha$$

and

$$\beta$$

.

Then,

$$\alpha + \beta = 1$$

$$\alpha\beta = 1$$

Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

.

$$\Rightarrow x^2 - 1x + 1$$

Therefore, the quadratic polynomial is

$$x^2 - x + 1$$

.

Ans:

Given:

$$-\frac{1}{4}, \frac{1}{4}$$

Let the zeroes of polynomial be

$$\alpha$$

and

$$\beta$$

.

Then,

$$\alpha + \beta = -\frac{1}{4}$$

$$\alpha\beta = \frac{1}{4}$$

Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

.

$$\Rightarrow x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$$

$$\Rightarrow 4x^2 + x + 1$$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

Ans:

Given:

$$4, 1$$

Let the zeroes of polynomial be

$$\alpha$$

and

$$\beta$$

.

Then,

$$\alpha + \beta = 4$$

$$\alpha\beta = 1$$

Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

.

$$\Rightarrow x^2 - 4x + 1$$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.